CONTROL SYSTEM IDENTIFICATION THROUGH MODEL MODULATION METHODS

ROBERT F. DREESEN
and
HARRY A. HOOVER

CONTROL SYSTEM IDENTIFICATION

THROUGH MODEL MODULATION METHODS

****

Robert F. Dreesen

and

Harry A. Hoover

CONTROL SYSTEM IDENTIFICATION
THROUGH MODEL MODULATION METHODS

by

Robert F. Dreesen

Lieutenant Commander, United States Navy

and

Harry A. Hoover

Lieutenant, United States Navy

Submitted in partial fulfullment of
the requirements for the degree of

MASTER OF SCIENCE
IN
ELECTRICAL ENGINEERING

United States Naval Postgraduate School
Monterey, California

1963

CONTROL SYSTEM IDENTIFICATION
THROUGH MODEL MODULATION METHODS

by

Robert F. Dreesen

and

Harry A. Hoover

This work is accepted as fulfilling

the thesis requirements for the degree of

MASTER OF SCIENCE

IN

ELECTRICAL ENGINEERING

from the

United States Naval Postgraduate School

ABSTRACT

Adaptive control system techniques have been utilized to investigate identification of the dynamic equations of operating control systems. The identification has been achieved by using model modulation techniques to drive dynamic models into correspondence with operating control systems. The system identification then proceeded from examination of the model and the adaptive loop.

The model modulation techniques applied to adaptive control systems are breifly discussed These techniques applied to simulation studies on an analog computer and a digital computer, and applied directly to a position servo-mechanism, are then discussed The discussion presents data showing the accuracy of identification of the dominant characteristics of the dynamic equations when the order of the equation is either known or approximated. It was found that these dominant roots could be found within an average accuracy of 5% for both complex phase angle and undamped natural frequency.

The authors wish to express their appreciation to Professor Richard C. Dorf of the U. S. Naval Postgraduate School, for the suggestion of the model modulation method and for his assistance during this investigation.

Table of Contents

List of Illustrations

List of Illustrations (continued)

TABLE OF SYMBOLS

C_s	Control system output
C_m	Model output
C/R	Closed loop transfer function
$G(s)$	Open loop transfer function
K	Generalized gain
m_c	Corrective signal derived from output
$m_{\dot{c}}$	Corrective signal derived from the derivative of output
p	Generalized s-plane pole
R	Input signal
(s)	Laplace variable
ζ	zeta - Damping factor of dominant complex roots
ω_n	Undamped natural angular frequency of dominant complex root

CONTROL SYSTEM IDENTIFICATION THROUGH MODEL MODULATION METHODS

Chapter I

INTRODUCTION

General

The last three decades have seen control system technology grow from a series of rules of application to a full regime encompassing most of the disciplines of engineering. The study of control systems has progressed deeply into analysis and synthesis of linear and non-linear control systems. There are, additionally, offshoots of basic systems with most exhaustive methodology: examples are the sampled data, hybrid analog-digital and multi-loop self adaptive systems.

The problem of control system identification led to some of the original investigation in the control field. Despite the increase of knowledge in the field of control engineering, system identification has remained a major problem.

The intent of this project was to investigate a method of control system identification utilizing techniques from the growing technology of the self-adaptive control system. One restriction was laid upon the project at its inception. This was that the method of system identification remain independent of the system to be identified Ideally then, an operating system should be identified solely through examination of the input and the response. To this end a control system identifier has been modeled and tested on an analog computer, through digital computer analysis, and upon a position control system.

Control System Identification

The characteristics and performances of the fundamental components of feedback control systems are generally amenable to systematic analysis. The control system, when composed of these fundamental components, may be easily synthesized and analyzed. Components, however, when removed from the theoretical state do not exhibit the invariant characteristics attributed to them. Furthermore, control systems are often synthesized on the basis of transfer function manipulation. This synthesis occurs under the assumption of negligible loading of one component system by another. This assumption is too often fallacious. In either or both of these situations the control system engineer is confronted with a control system which does not conform to previous analysis. And if this analyzed control system is a portion of some larger process, it may be necessary to reevaluate the control characteristics and performance, in other words to identify the system.

In some extremely variant systems the processes of system identification are utilized to serve as controllers in a self-adaptive control system. The most notable of these cases are the self-adaptive control systems currently in use and being further developed for aircraft installation. The NASA X-15 is an example. In this installation the response characteristics are examined and the results are used to vary flight control loop gain to maintain near invariant control response. The equipment used for this process is quite complex, but is justified by the operating environment of the vehicle.

For less complex control systems or in less complex environments, the problem of control can be solved by more conventional means, without recourse to dynamic self-adaptive system technology. These less complex systems require accurate identification of the process dynamics. Once the transfer function of a process has been evaluated this evaluation can serve as a basis for design of the remainder of a system or for readjustment of the process through compensation techniques.

Identification Methods

Truxal writing in reference (1) has placed the methods of control system identification into four broad categories:

a. Parameter identification.

b. Frequency response methods.

c. Impulse response methods.

d. Direct evaluation of system differential equations of motion.

Some of the difficulties in one-by-one determination of model parameters have been mentioned. Additionally, this method is possible only when the detailed operation of a process is reducible to fundamental differential equations in which the primary parameters may be included and the negligible ones discarded. This ideal situation rarely exists for complex control of complex processes.

As frequency response techniques are well-known, a description of them has been omitted. These procedures are extremely important in general control design. This is the most common method of identifying the transmittive dynamics of a control process. Many working control systems however, cannot be subjected to variable frequency oscillations for reasons of system complexity, operating economy, or

because of the extensive time and effort required for determination of the gain and phase characteristics. Two additional problems occur. First, in some circumstances, frequency response testing should not be superimposed on regular operation, because testing would interfere with designed operation. Secondly, a working system is subjected not only to its driving input, but also to noise occuring during the operating process. The extraneous disturbances of normal operation would make the interpretation of frequency test results difficult.

The impulse response can serve to completely identify any linear time-invariant two port system. The various impulse response techniques are utilized in many of the more sophisticated adaptive control systems. Truxal has cited three primary techniques for measuring impulse response. The first and most direct method is to excite a system with an impulse and measure the response. Second, the system can be excited by a known signal. For an initially inert system, the impulse response $g_{(t)}$ can be determined from the input, $r_{(t)}$, and output, $g_{(t)}$, through the convolution integral

$$c_{(t)} = \int_0^t g(\tau)\, r(t-\tau)\, d\tau \qquad 1.1$$

Finally the impulse response can be measured by application of a small random signal to the process. If white noise is used as the input, the cross correlation function between the output and the noise is the impulse response.

The attempt to utilize the impulse response in identification is complicated by three problems. Noise has a much more delerterious

effect on the output than in the case of frequency response tests. Nonlinearities are much more difficult to detect than in frequency response tests. Finally the mechanization necessary to determine the impulse response is almost prohibitive for use in system identification. When the impulse response method is used in adaptive system control the engineer is generally interested not in the full identification, but in identifying only some characteristic of the response. Quantitative measurements are made to serve as sufficient identification to allow adaptive correction. As an example, the number of oscillations during a specific period of a system response would identify the impulse response. These approximations are not sufficient for full identification, however.

The evaluation of the differential equation for a linear process offers an additional method of identification. The differential equation of a process might be expressed as an equation of input $r_{(t)}$ and output $c_{(t)}$ in the following form:

$$\frac{dc^N}{dt^N} + a_{N-1}\frac{dc^{N-1}}{dt^{N-1}} + \circ\circ\circ + a_1\frac{dc}{dt} + a_o = b_M\frac{dr^M}{dt} + \circ\circ\circ + b_o r_o \qquad 1.2$$

The a_o term may be evaluated from the nature of the zero frequency or steady state gain. If the order of the differential equation is known, and the nature of the right hand side is known, the other a_{N-1} coefficients may be evaluated by generalizations such as minimizations of square of the error e, where e is defined:

$$e = \frac{dc^N}{dt^N} + \circ\circ\circ + a_1\frac{dc}{dt} + a_o c - b_o r \qquad 1.3$$

The a_{N-1} coefficients are evaluated through use of this error criterion by generation of successive differentiations of the response. Unfortunately, searching over an (N-1) space for a minimum is a problem of potential difficulty. General problems also arise in settling at local minimums and in error system stability.

Many of the problems of systematic determination of the differential equation coefficients are alleviated by the use of system-model reference. This is a natural variant to the adaptive control system technology. The exploitation of this method involves utilization of the difference between system and model response to the forcing input. This difference has been handled by several methods in present technology. In general self adaptive processes, the difference as operated upon by an adaptive technique is used to drive the system into operating correspondence with the invariant response of the model.

This investigation was based on a variation of this method. During this investigation adaptive techniques have been used to drive the model into operating correspondence with the operating system. The characteristics of the model differential equation have been modified by the adaptive loop. These characteristics then have been used to accurately define the coefficients of the system differential equation.

Chapter II

MODEL REFERENCE ADAPTIVE SYSTEMS

Introduction

A major advantage of the model reference adaptive system is that direct measurement of the operating system differential equation parameters is not required. This factor results in considerable economy in mechanization of the adaptive components. As previously mentioned there are in existence various methods of self adaptive control. However, in many of these systems the adaptive loop mechanization requires the assumption of the second order nature of the system. An example of this is Osder's system detailed in reference (1). For this system, the sign reversals in the output oscillations are counted and the count used to correct the second order zeta. Most of the impulse response self adaptive systems also require assumption of a basically second order system to keep the equipment within allowable limits. The model reference system relieves this restriction. The model reference adaptive system can be utilized for higher order systems about which the second order assumption is invalid. Additionally, current literature presents several examples in which model reference allows the control of several parameters within the operating system.

References (1) and (2) present several methods of model reference adaptive control systems which have been proposed or developed. The variants of these systems include the Ham-Lang conditional control system, which in analysis is passive adaptation through model reference.

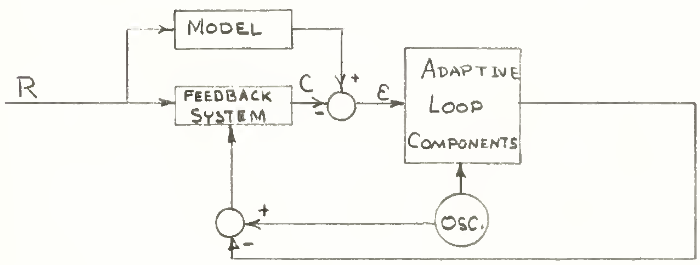
The other extreme is perhaps, Braun's method, which utilizes an orthonormal spectrum analyzer to solve the identification problem.

Parameter Perturbation

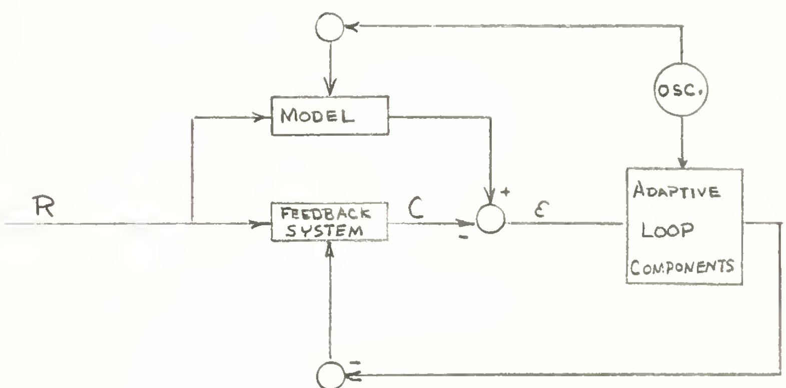
Perhaps the most straightforward adaptive mechanization results from the use of model reference with perturbation techniques. McGrath, Rajaraman and Rideout in reference (3) have proposed and examined various parameter perturbation model reference schemes. Figure 1, taken from their discussion presents two variants of the parameter perturbation scheme.

Fig. 1

PARAMETER PERTURBATION METHODS



(a)



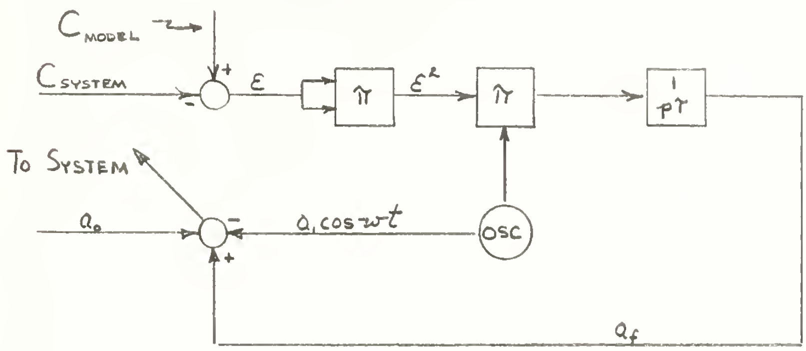
(b)

The adaptation processes in these systems are driven by the error, ε, between the model and system. This error is used to form a performance

criterion, which for this example was integral of error squared, $\int e^2 dt$. This criteria contains a component of the frequency at which a parameter of the system in Fig. 1(a) is being perturbed. The amplitude and phase of this component give the magnitude and sign of the signal obtained through perturbation and integration. This signal is then fed back negatively to reduce the error. The adaptive loop mechanization is illustrated in Fig. 2.

Fig. 2

PARAMETER PERTURBATION ADAPTIVE LOOP



For analog simulation electronic multipliers have been utilized for parameter correction insertion. In operating systems servo-multipliers could be utilized.

The system of Fig. 1(a) suffers from the disadvantage that the perturbation appears in the output. The system of Fig. 1(b) avoids this problem through model parameter perturbation. This second method, however, requires that the model be nearly the same form as the system. Both methods, moreover, require phase correcting networks to correct the phase shift between parameter perturbation signal in the model and the oscillator signal. Through utilization of further model forms with parameter perturbation the authors demonstrate that it is possible

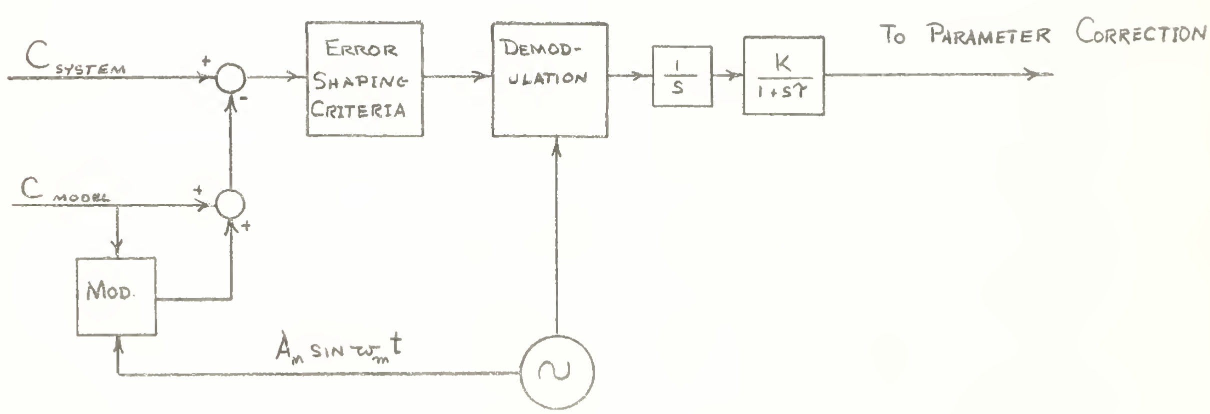
to prepare a configuration which is both system and signal adaptive, using multiple models with perturbation of either model or system parameters. Rajaraman in a later article, Ref. 4, has shown the detailed operation of the signal and system adaptive process. In summary, this signal and system adaptive process requires two models and two adaptive loops. The two models are of different nature. The first is an ideal version of the process, and the second is of the same order as the process. The second model is optimized for the input by the first adaptive loop operating upon the difference between the output of the two models. The second adaptive loop causes system parameters to follow the optimized second model parameters, and at the same time corrects for disturbances.

Model Output Modulation

Dorf and Byers have shown the feasibility of the model reference self adaptive system using output modulation techniques. The analysis of the techniques and a discussion of experimental results are found in Ref. 5. They are briefly summarized here. Fig. 3 presents the block diagram of the system.

Fig. 3

MODEL MODULATION ADAPTIVE LOOP



In this adaptive loop, the output error is:

$$E_{(t)} = C_s\ (t) - C_m\ (t) \quad \left[1 + A \sin \omega_m\ t\right] \tag{2.1}$$

For this example error squared has been used as the criterion. Thus, after being shaped and demodulated a phase-sensitive d.c. signal is obtained. When this signal is subjected to integration and filtering, the open loop output of the adaptive circuit becomes:

$$A^2\ C_m \quad (C_m - C_s)\ t \tag{2.2}$$

The adaptive circuit loop was closed by this signal being inserted into the system as a driving function, to correct a system parameter. The result of this multiloop circuit is a complex, non-linear system which has adaptive characteristics to maintain an approximately invariant system transfer function. In functional notation the system output might be described as a function of the following variables:

$$C_{(t)} = f \quad (K_{Ad},\ \omega_i,\ \left|R\right|,\ \tau_f,\ T,\ M)$$

K_{Ad} = adaptive loop gain

ω_i = frequency of input signal

R = magnitude of input signal

τ_f = filter network time constants

T = system transfer function, uncorrected

M = model transfer function

The adaptive loop filter was designed to filter out the normal system operating signals. The speed of adaptation depends partially on the filter transfer function. The gain and lag of the filter influence the magnitude of the adaptive response and the input signal contami-

nation. One important feature of this method is that an absolutely constant frequency oscillator is not required. The operation of the adaptive loop is not dependent upon frequency stability of the modulating frequency. Furthermore, as there are no significant time constants between modulation and demodulation, the frequency of modulation is limited only in bounds at the lower frequency by the necessity of being an order above the input signal frequency. Dorf and Byers further investigated a two parameter correction situation, driving one adaptive loop with output and the second adaptive loop with the derivative of output. In their example the system open loop transfer function was:

$$G(s) = \frac{K_1 \omega_N^2}{s\left(s^2 + 2\zeta\omega_N + \omega_N^2\right)} \qquad 2.3$$

The model open loop transfer function was similar. The first adaptive loop was driven by $\dot{C}_m$ and $\dot{C}_s$ to correct $(K_1 + \Delta K)$, the open loop gain. The second adaptive loop was driven by C_m and C_s to control the inner loop damping $(2\zeta\omega_n + \Delta 2\zeta\omega_n)$. The authors concluded that a two parameter self adaptive system of this kind was feasible. A major restriction was found to exist if the two parameters being adapted were not independent, and the adaptation would be successful only if the range of required adaptation was restricted.

Application of Model Modulation

To verify the results of model modulation, and to obtain experience with the characteristics of the method, the method was simulated on a Donner 3100 analog computer. Initial investigation concerned

the use of a second order system with one variable parameter being adapted to follow a second order model. The model and the system were ordered for an open loop transfer function:

$$G(s) = \frac{\omega_N^2}{s(s+2\zeta\omega_N)} = \frac{100}{s(s+10)} \tag{2.4}$$

The parameter drift then is represented as

$$G(s) = \frac{\omega_N^2}{s(s + 2\zeta\omega_N + n)} \tag{2.5}$$

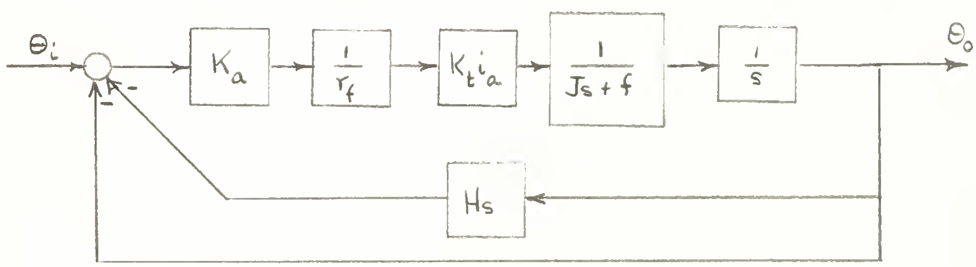
where n was a step parameter drift. The input was also of various wave forms of frequency less than the bandwidth. The adaptive system open loop is represented as:

$$G(s) = \frac{\omega_N^2}{s(s + 2\zeta\omega_N + n - m_c)} \tag{2.6}$$

where m_c is the correction derived from the output driven adaptive loop. This system might be considered to simulate a second order position servo as displayed in Fig. 4.

Fig. 4

SECOND ORDER SERVOMECHANISM



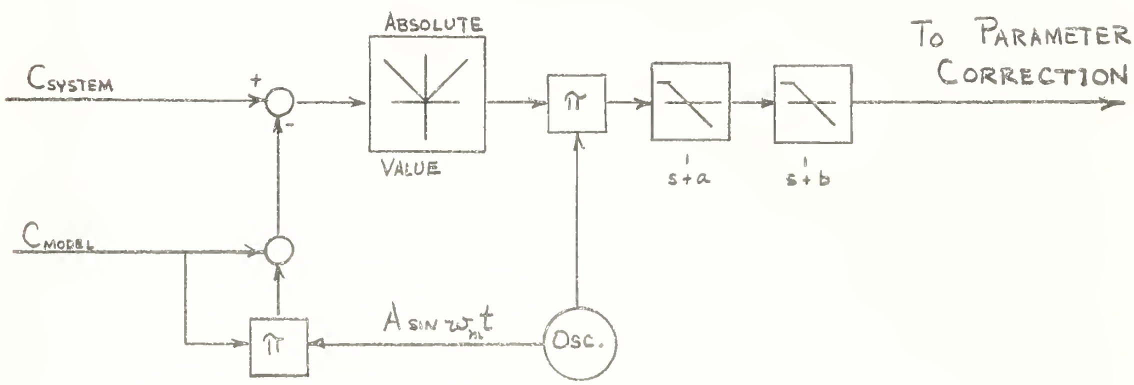
Constant Armature Current D.C. Motor Drive

Thus the variable drift simulated a system with varying or unknown friction(f), a system with constant gain but varying velocity error coefficient. The correction would be inserted by varying tachometer gain feedback magnitude.

For operational simplicity the error criterion was altered from the integral of error squared, ISE, to the integral of absolute error or IAE. Additionally, the modulation and demodulation was accomplished through use of electronic multipliers. Incidental noise problems with these multipliers was such that the pure integration followed by a filter was removed from the adaptive loop, and instead a double filter network substituted. The alteration is shown symbolically in Fig. 5.

Fig. 5

MODIFIED MODEL MODULATION LOOP



Satisfactory results were achieved with this circuit with the filter break points a = .1, and b = .1. The full computer diagram is shown in Fig. 6. The value of the gain of the adaptive loop for the circuit of Fig. 6 was 3. Fig. 7 presents the output data for this same circuit, illustrating adaptation for various parameter drift conditions.

Additional investigation was made of the control of a third order system with one variable parameter by a second order model. The third order system investigated open loop transfer function was

$$G(s) = \frac{K_p \; 100}{s\,(s + K_p)\,(s + 10 + n - m_c)} \qquad 2.7$$

The model utilized was that of the previous example. The analog computer setup was that of the transfer function format similar to Fig. 6, with an additional function $\frac{K_p}{s + K_p}$ installed. The adaptive loop was unchanged. This system then simulated a third order control system as illustrated in Fig. 8.

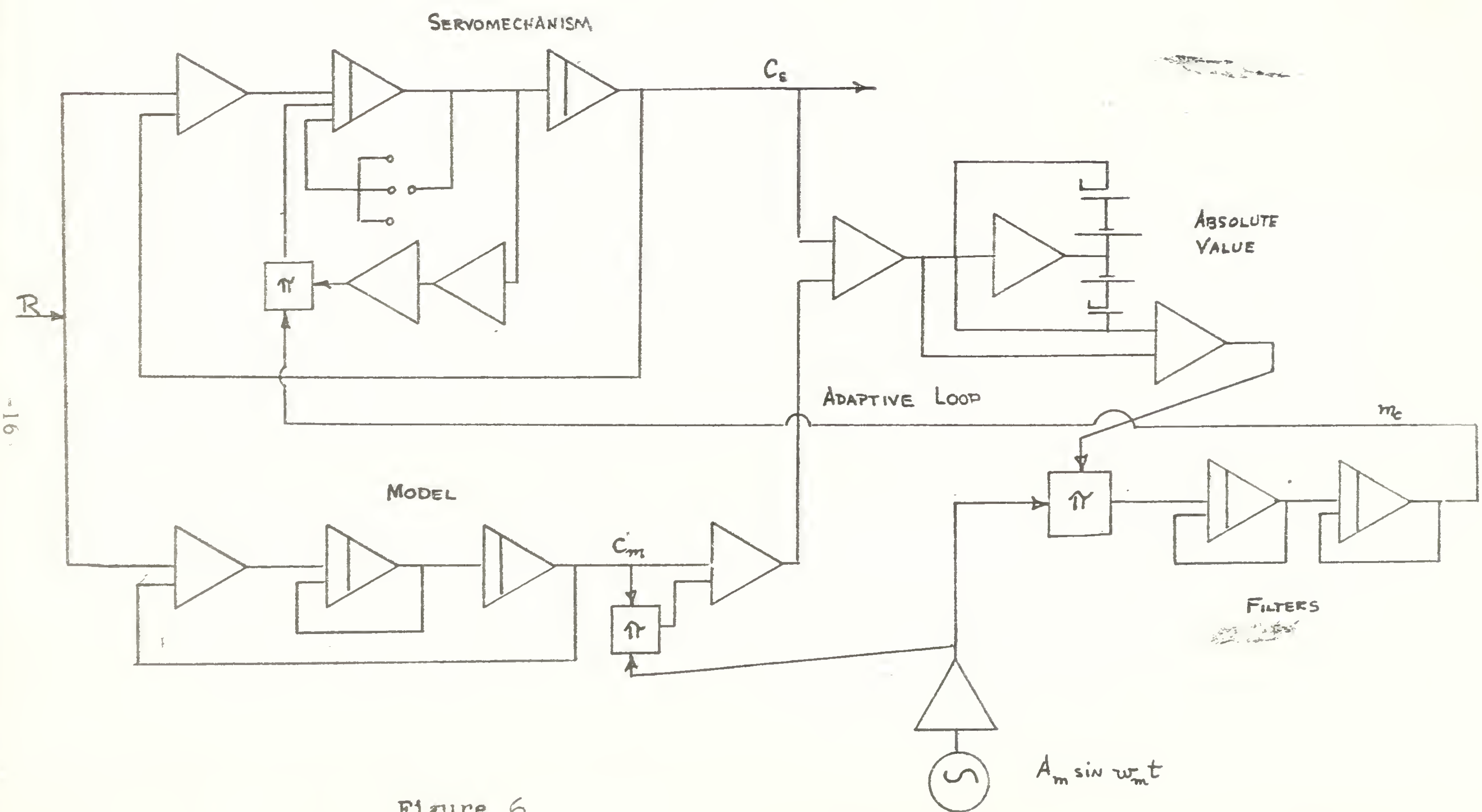
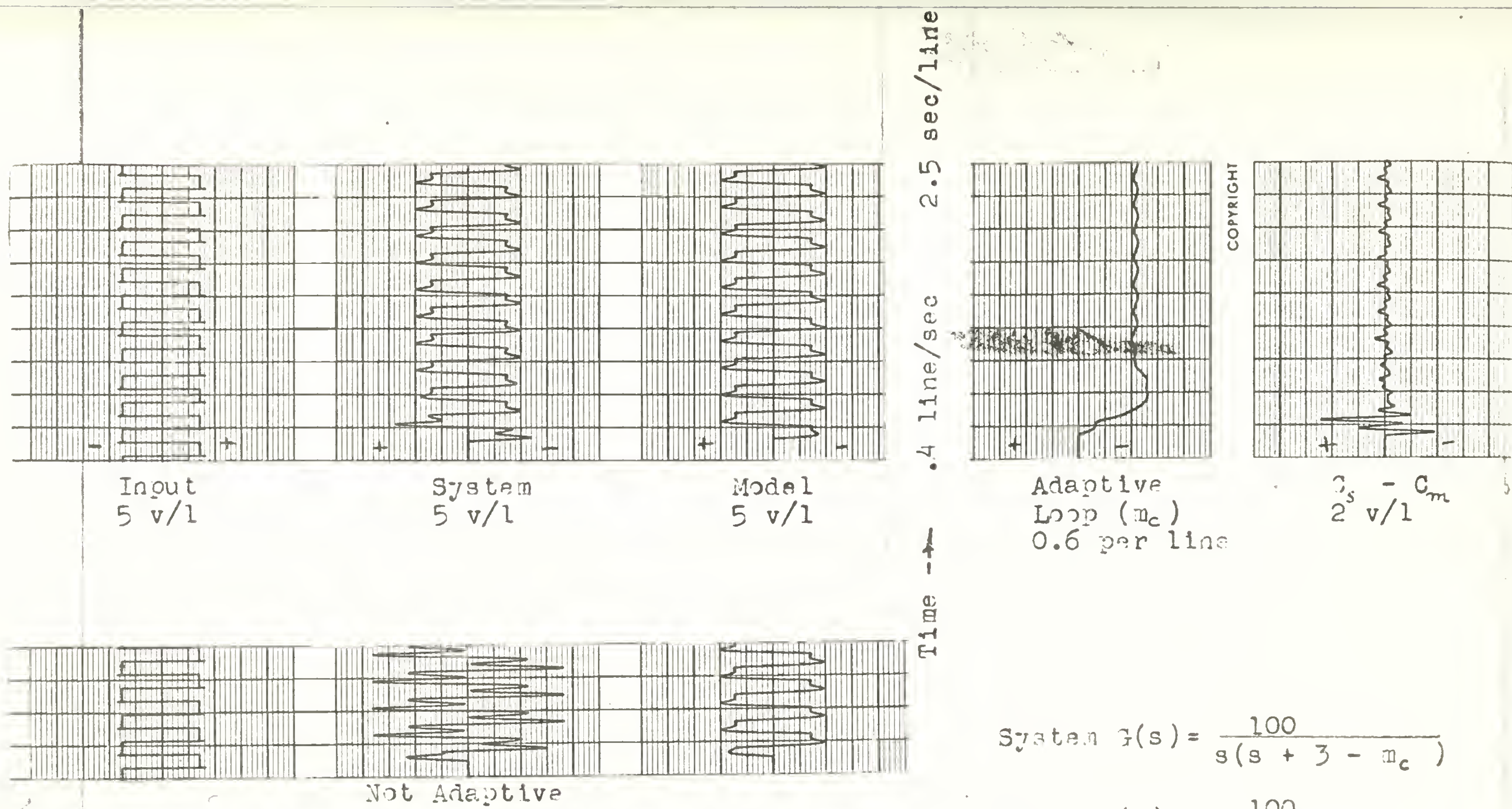


Figure 6

Analog Computer Diagram for Single Parameter
Adapting Servomechanism

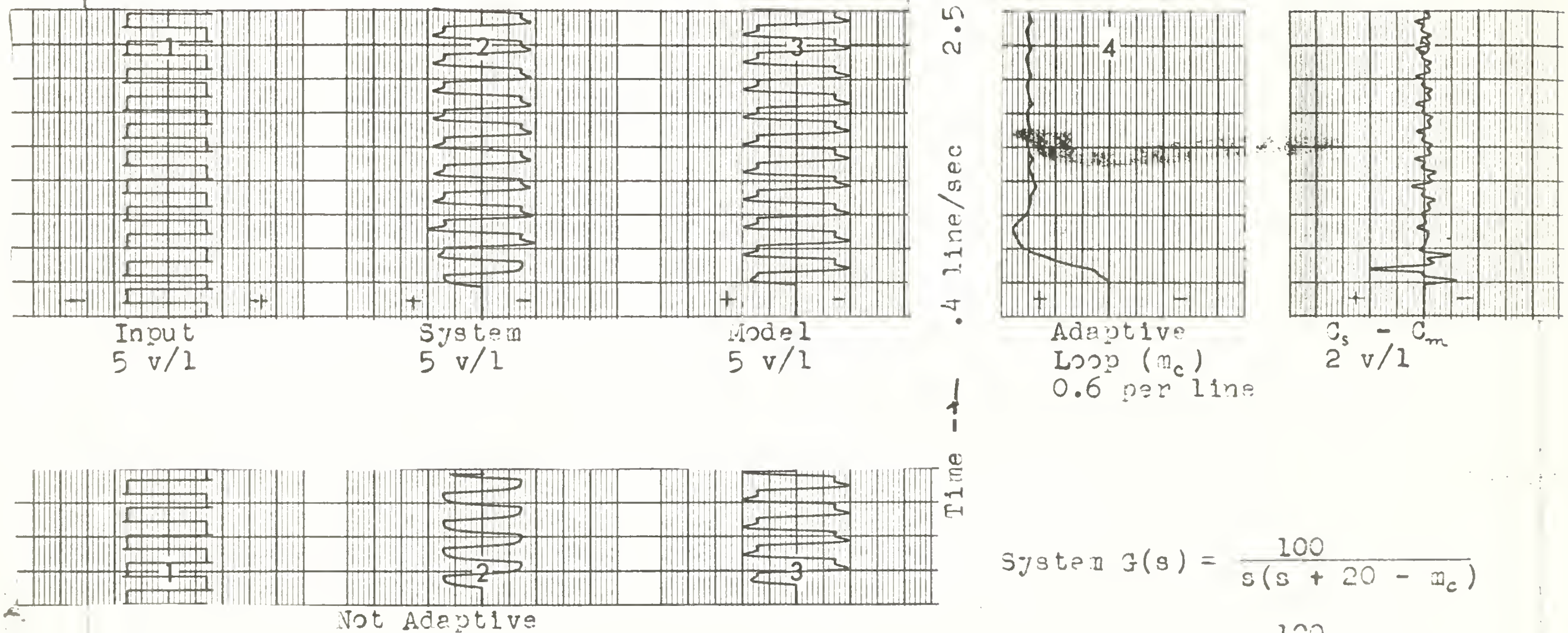


$$\text{System } G(s) = \frac{100}{s(s + 3 - m_c)}$$

$$\text{Model } G(s) = \frac{100}{s(s + 10)}$$

Figure 7a

Second Order System Adapting from Zeta = .15 to Zeta ≈ 0.5



$$\text{System } G(s) = \frac{100}{s(s + 20 - m_c)}$$

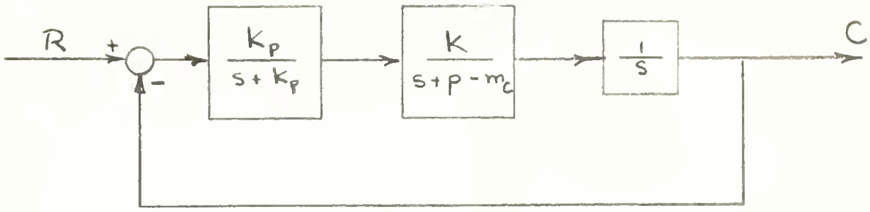
$$\text{Model } G(s) = \frac{100}{s(s + 10)}$$

Figure 7b

Second Order System Adapting from Zeta = 1.0 to Zeta ≈ 0.5

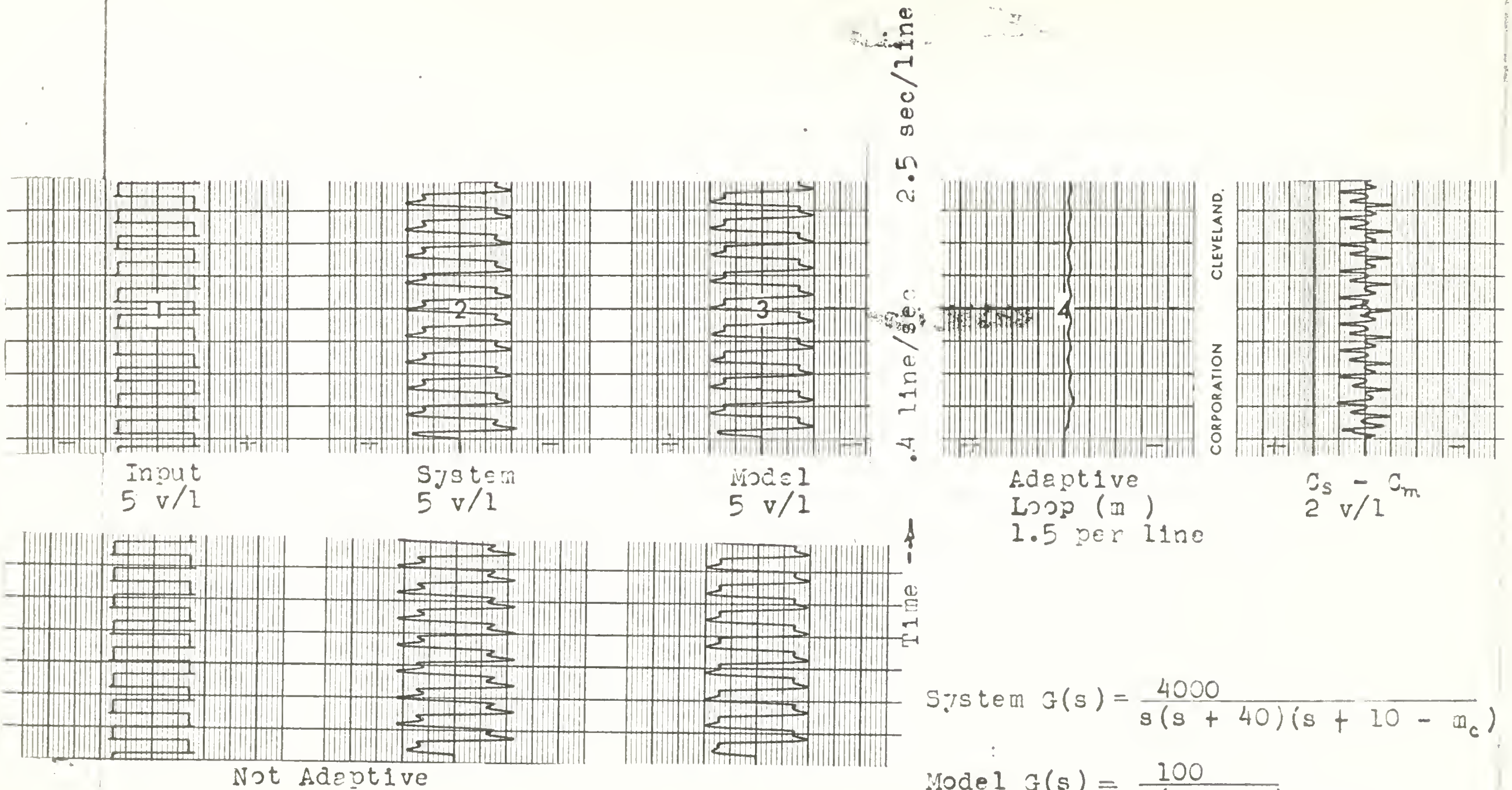
Fig. 8

THIRD ORDER SYSTEM



For this situation the variable parameter simulated would be p, with the correction being inserted through a small servo varying m_c. The adaptation was satisfactory for $K_p = 40$, when $p = 10$. Adaptability became difficult for $K_p < 40$ for large parameter shift. Fig. 9 presents the output data for this circuit for a representative $K_p = 40$, under various parameter drift conditions.

As a final illustration of the model modulation method, cancellation techniques were used for system control. The process is best illustrated in Fig. 10.

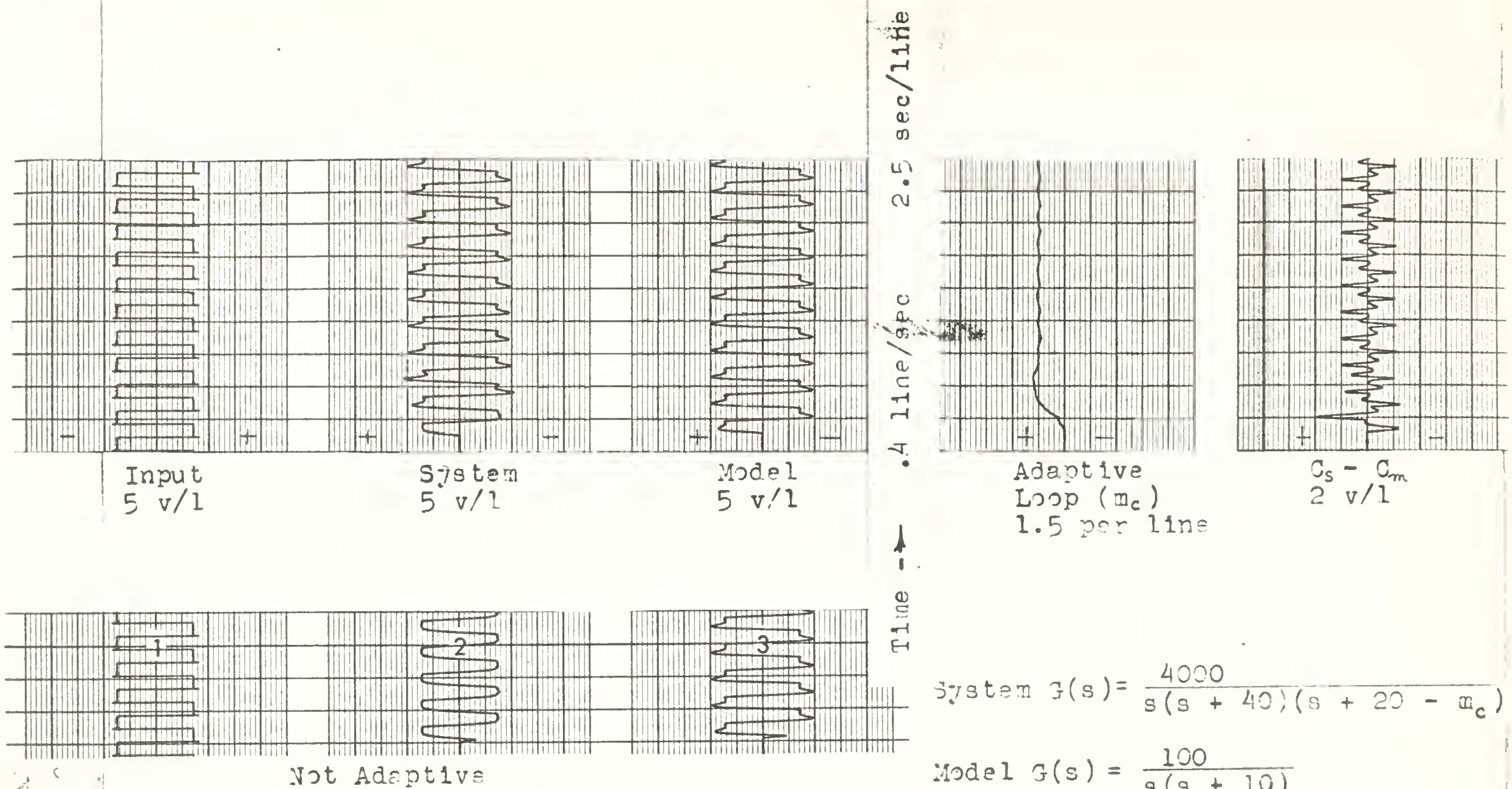


$$\text{System } G(s) = \frac{4000}{s(s + 40)(s + 10 - m_c)}$$

$$\text{Model } G(s) = \frac{100}{s(s + 10)}$$

Figure 9 a

Third Order System Adapting to Second Order Model

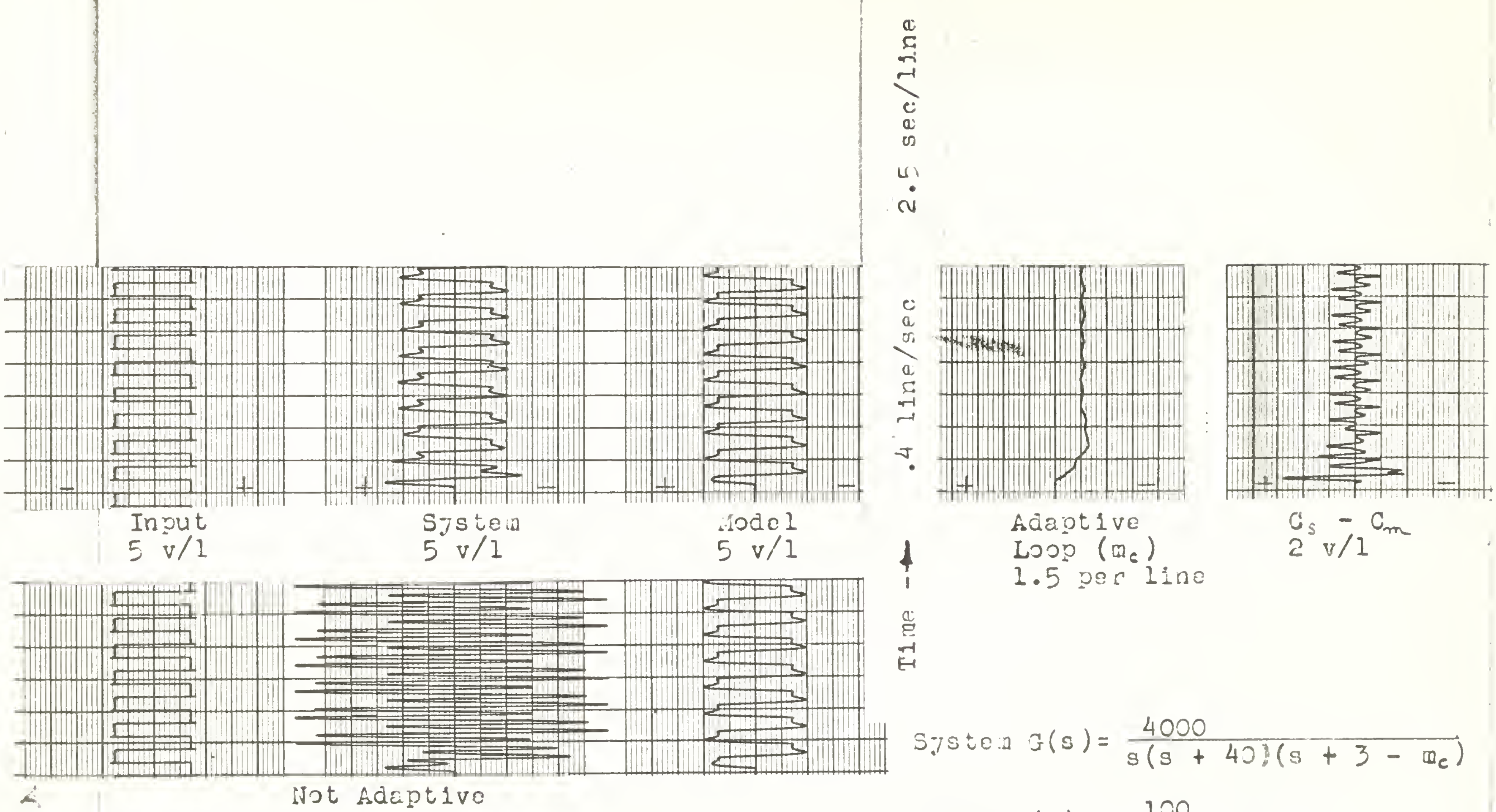


$$\text{System } G(s) = \frac{4000}{s(s + 40)(s + 20 - m_c)}$$

$$\text{Model } G(s) = \frac{100}{s(s + 10)}$$

Figure 9 b

Third Order System Adapting to Second Order Model



$$\text{System } G(s) = \frac{4000}{s(s+40)(s+3-m_c)}$$

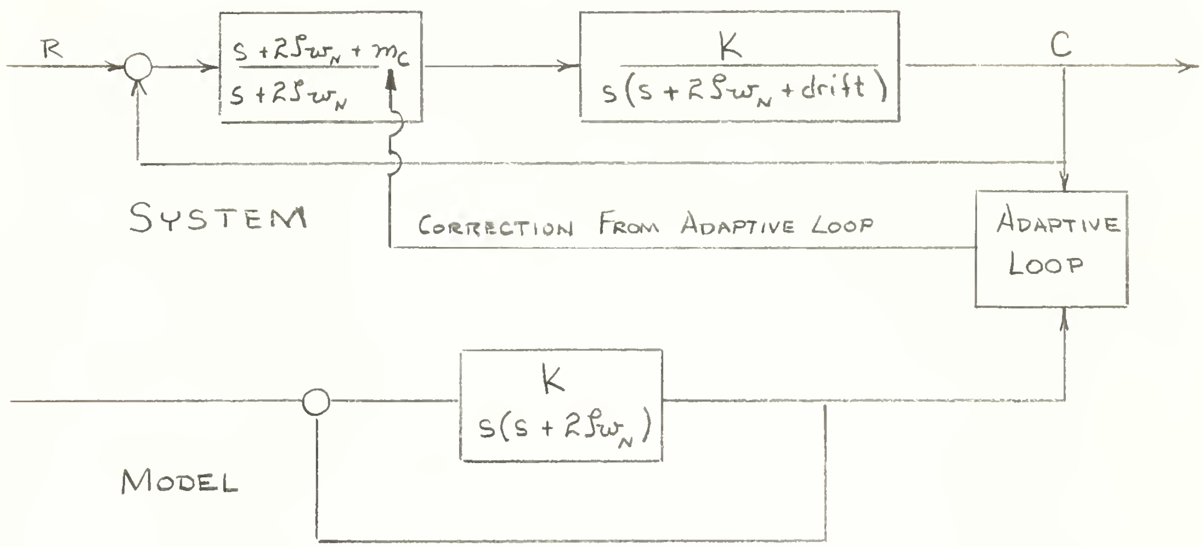
$$\text{Model } G(s) = \frac{100}{s(s+10)}$$

Figure 9 c

Third Order System Adapting to Second Order Model

Fig. 10

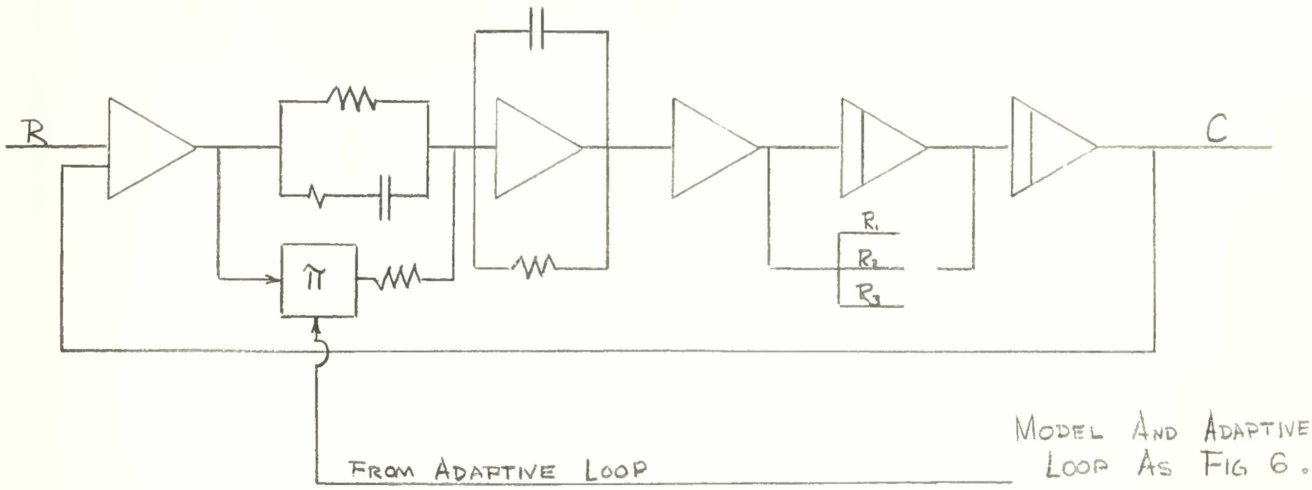
CONTROL SYSTEM CASCADE COMPENSATION ADAPTATION



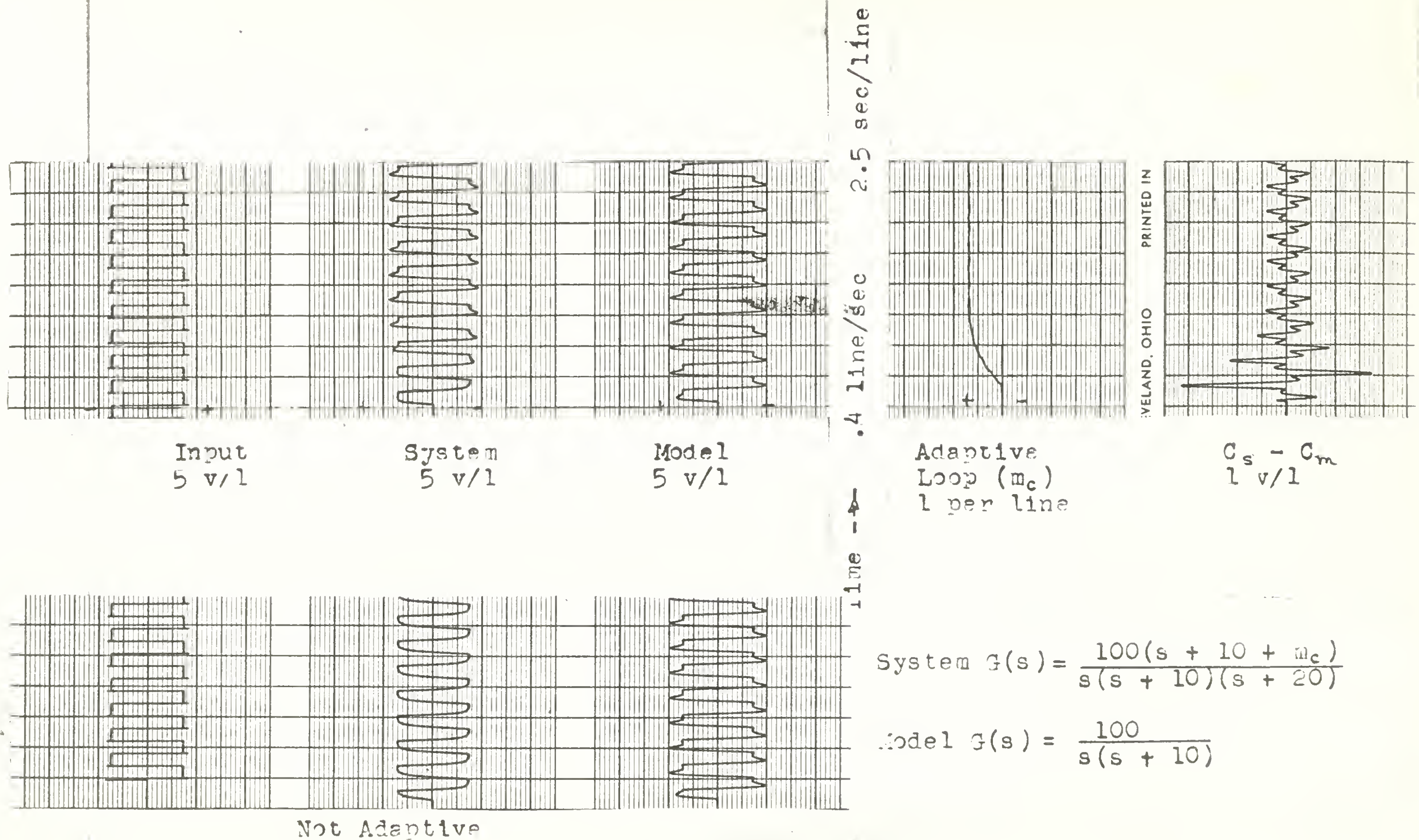
This technique is most useful when the operating system pole is not amenable to correction because of the physical characteristics of the system. In the operation illustrated by Fig. 10 zero drift caused compensator cancellation. When drift was introduced the correction forced the zero of the compensator to cancel the system pole, and the desired system operation was restored. The analog computer operating schematic is presented in Fig. 11. For operational simplicity the drift was inserted as a step change in the system pole. The computer output data is presented in Fig. 12, where the successful adaptation of the system is well illustrated.

Fig. 11

ANALOG COMPUTER SIMULATION OF COMPENSATION ADAPTATION



The results of these simulations of the adaptive system were considered to satisfactorily demonstrate the feasibility of the method and to serve for participant training. The analytical proof of the correctness of the alterations to the initial formulation, IAE for ISE as criterion, and filter instead of pure integration, have been neglected. The demonstrated accuracy of the adaptation was felt to be sufficient proof of the method.

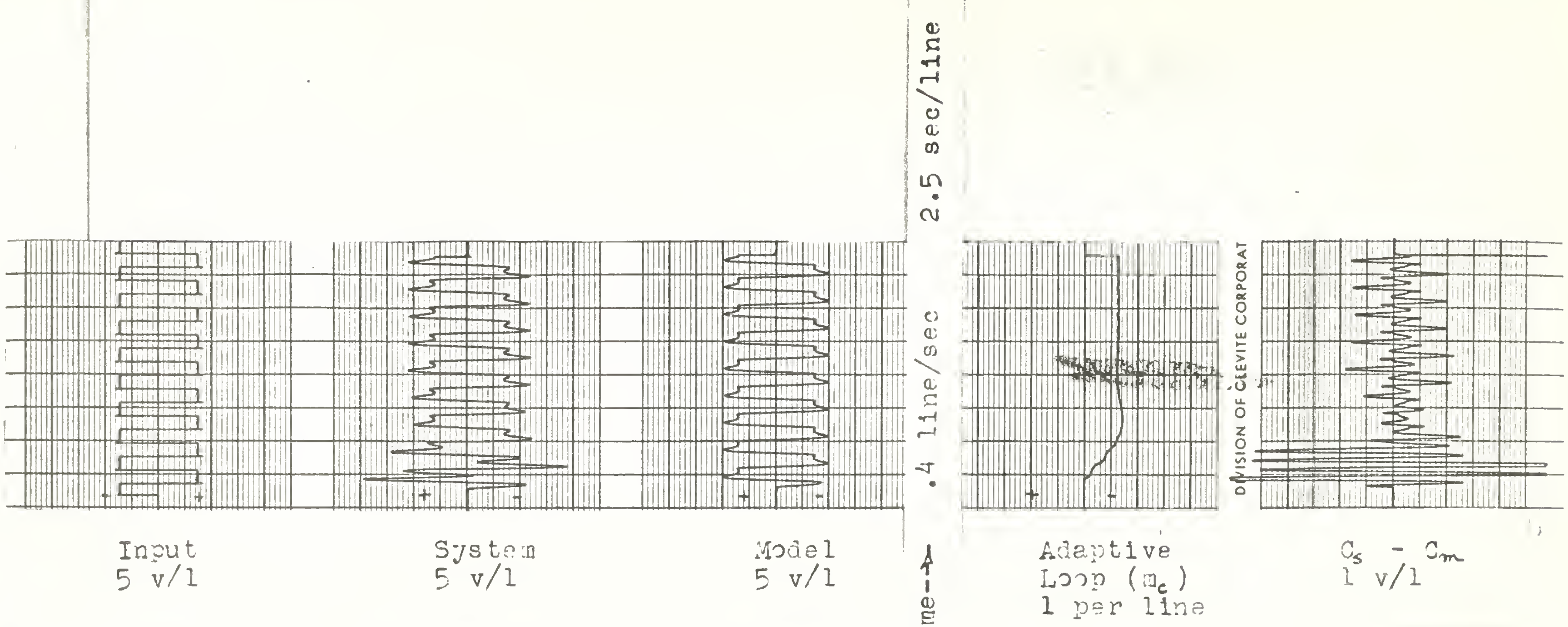


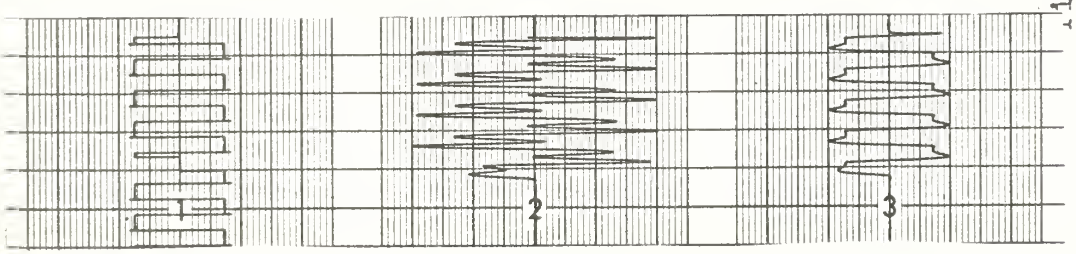
$$\text{System } G(s) = \frac{100(s + 10 + m_c)}{s(s + 10)(s + 20)}$$

$$\text{Model } G(s) = \frac{100}{s(s + 10)}$$

Figure 12a

Second Order System Adapting from Zeta = 1.0 to Zeta ≈ 0.5





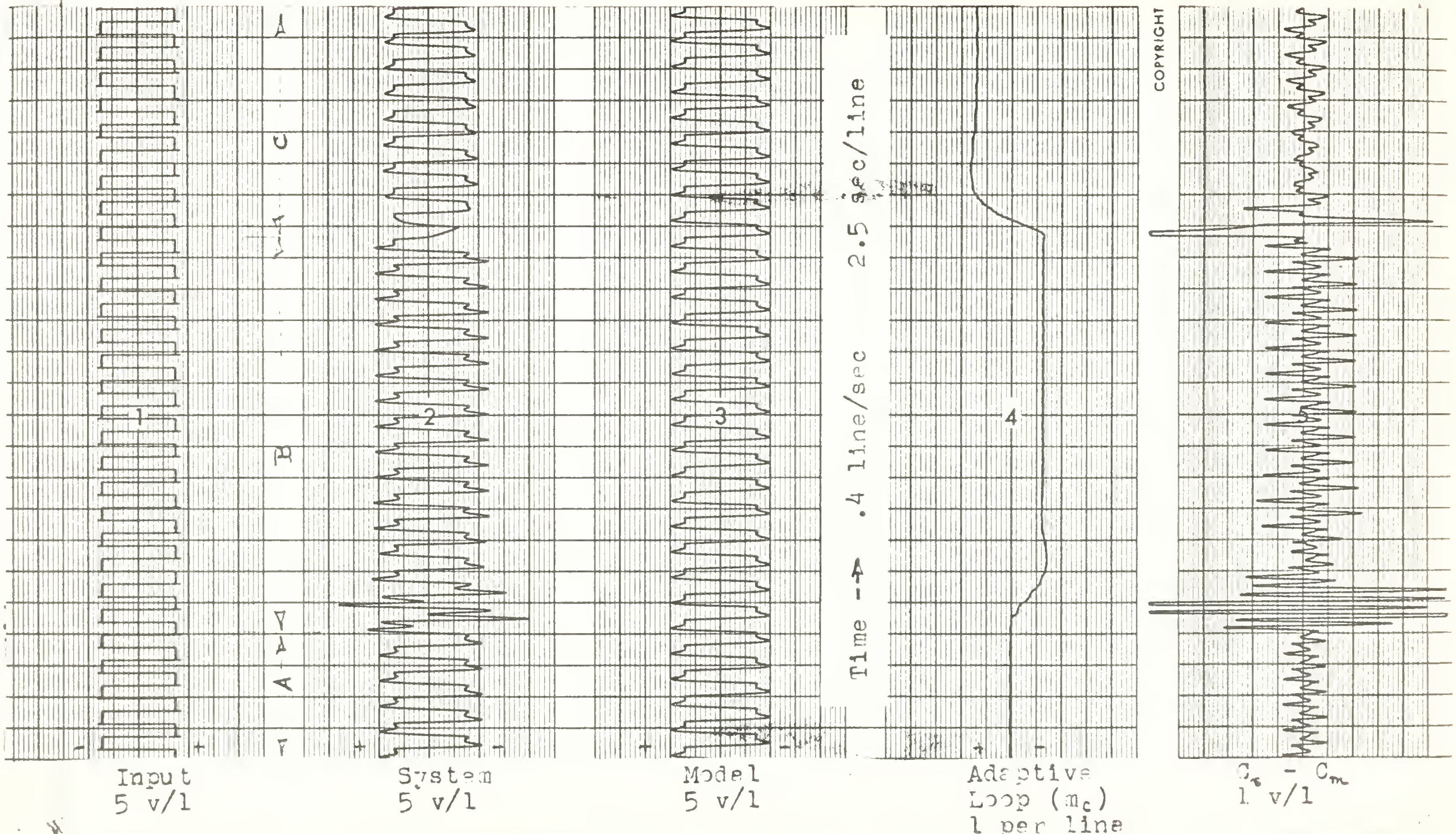
$$\text{System } G(s) = \frac{100(s + 10 + m_c)}{s(s + 10)(s + 3)}$$

$$\text{Model } G(s) = \frac{100}{s(s + 10)}$$

Not Adaptive

Figure 12b

Second Order System Adapting from Zeta = .15 to Zeta ≈ 0.5



Model $G(s) = \frac{100}{s(s + 10)}$

System $G(s) = \frac{100(s + 10 + m_c)}{s(s + 10)(s + 10)}$ (A)

$= \frac{100(s + 10 + m_c)}{s(s + 10)(s + 3)}$ (B)

$= \frac{100(s + 10 + m_c)}{s(s + 10)(s + 20)}$ (C)

Figure 12 c

Second Order System; Adaption to Zeta Steps

Chapter III

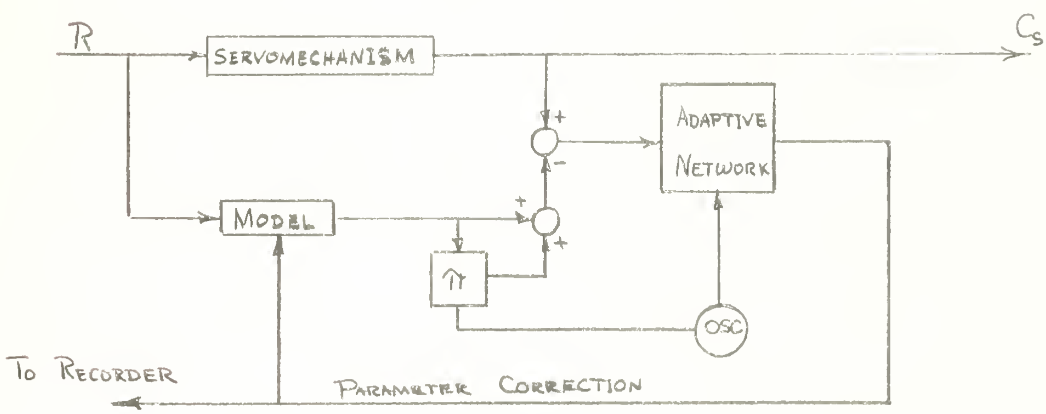
IDENTIFICATION THROUGH MODEL MODULATION

The methods of successful adaptive control through model modulation were directly applicable to the identification of operating systems. The requirements initially set were that the identification should proceed utilizing only the operating system input and output without disturbing the system operation. To utilize the model modulation method, the model would be placed in parallel operation with the system. The adaptive loop output would be applied to the model parameters to cause the model to adapt to the system. Thus, knowledge of the unadapted model governing equations combined with the magnitude of the adaptive loop output would serve to define the system.

This method is shown in block diagram form in Fig. 13.

Fig. 13

MODEL MODULATION WITH MODEL ADAPTATION



This method of system identification does not relieve the engineer of the necessity of system analysis. The form of the transfer function, the order of the operating equation, and the general magnitude of the poles and zeros of the system must be known in order to synthesize

the model. Ideally, one or two critical parameters whose magnitude may not be susceptible to analysis may be estimated, then measured or identified by the model modulation. These limitations should be expected from any adaptive system. The knowledge required to build an adaptive loop which would assume no knowledge of the system, and then synthesize perfect identification, with sufficient economy to be practical, is beyond the state of the art.

The standard second order servo or feedback system equation is formulated in transfer function terms as:

$$\frac{C(s)}{R(s)} = \frac{\omega_N^2}{s^2 + 2\zeta\omega_N + \omega_N^2} \qquad 3.1$$

For this system the standard open loop transfer function is:

$$\frac{C(s)}{\mathcal{E}(s)} = G(s) = \frac{\omega_N^2}{s(s + 2\zeta\omega_N)} \qquad 3.2$$

The response of the second order system is completely described by the two variables, zeta and ω_n. A higher order system, or any system with closed loop transfer function zeros, naturally, has additional significant variables. If, however, it is considered that there are two variables of primary interest, the higher order open loop transfer function may be written as:

$$\frac{C(s)}{\mathcal{E}(s)} = G(s) = \frac{\omega_N^2 K \prod_{i=1}^{m} (s + z_i)}{s(s + 2\zeta\omega_N) \prod_{K=1}^{o} (s + p_K)} \qquad 3.3$$

where ω_n and zeta are these variables.

In the development of model modulation system identification method two major assumptions were made. The first of these is.

1. That in the operating system to be identified there will be but two unknown parameters. Thus the system must be second order, a completely dominant second order approximation of a higher order system, or a higher order system which has the other parameters fully identified. If the latter is the case, the format of the preceding equation will be used to describe the system.

This first assumption limited the parameter adaptation problem to the two variables zeta and ω_n.

Examination of any physical system reveals that zeta and ω_n may or may not be functionally independent. The simple second order system detailed in Fig. 5 serves as an adequate example. The closed loop transfer function is:

$$\frac{C(s)}{R(s)} = \frac{\dfrac{K_a K_t i_a}{rJ}}{s^2 + s\left(\dfrac{f}{J} + \dfrac{HK_a K_t i_a}{rJ}\right) + \dfrac{K_a K_t i_a}{rJ}} \qquad 3.4$$

In this system variation of the friction(f) will yield an independent zeta and ω_n. Variation of all other factors will result in zeta and ω_n being different functions of the same variable, and thus not independent. Thus zeta and ω_n may be independent or both different functions of the same variable.

The adaptive method that has been discussed to this point has dealt with but one variable. Since the interdependence of zeta and ω_n is at the most one of their being two different functions of one factor, it is reasonable to expect that two adaptive loops will be required.

The second major assumption required depends heuristically on this discussion of the functional relationship of zeta and ω_n. This assumption is:

II.

That model may be adapted to the system through two separate adaptive loops. The first of the loops will operate upon the system and model output to develop the first corrective factor. The second adaptive loop will operate upon the derivative with respect to time of the system and model output to develop the second corrective factor.

The remainder of this report is devoted basically to showing that these two assumptions are justified.

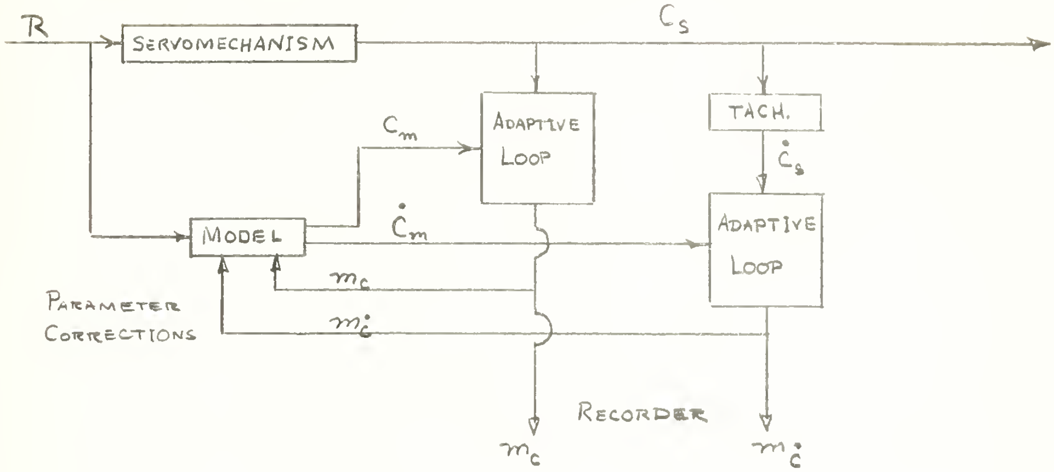
Proceeding on these assumptions the investigation utilized models of the following general form:

$$G(s) = \frac{K\left(\omega_{N_1}^2 - m_C\right)\prod_{i=0}^{i=N}\left(s + z_i\right)}{s\left(s + 2\zeta_1\omega_{N_1} + m_{\zeta}\right)\prod_{K=0}^{m}\left(s + p_K\right)} \qquad 3.5$$

A schematic of the operation of these loops as tentatively applied to system identification is illustrated in Fig. 14.

Fig. 14

SCHEMATIC OF TWO PARAMETER CORRECTION



With this model the characteristic equation of the operating system could then be easily detailed. For the model with the form of equation 3.5 and unity feedback, the closed loop transfer function becomes:

$$\frac{C(s)}{R(s)} = \frac{K\left(\omega_{N_1}^2 - m_c\right)\prod_{i=0}^{N}\left(s + z_i\right)}{s\left(s + 2\zeta_1\omega_{N_1} + m_{\dot{c}}\right)\prod_{K=0}^{m}\left(s + p_k\right) + K\left(\omega_{N_1}^2 - m_c\right)\prod_{i=1}^{N}\left(s + z_1\right)} \qquad 3.6$$

This function may then be used to accurately detail the system differential equations.

Only limited single parameter, single adaptive loop correction investigation was made. Primary investigation concerned the two parameters of the previous equation as in Fig. 14.

Fig. 14 also illustrates the visualized mechanization of the model modulation identification system. Potentially the model and the adaptive

loops would be constructed through use of standard operational amplifier networks. Thus any moderate sized analog computer operating in conjunction with the scaled input and output of a physical system would serve as an identifier.

Chapter IV

ANALOG COMPUTER INVESTIGATION

Model Modulation Identification of an Equation of Known Order

The primary investigation was centered around use of the analog computer. The investigation concerned use of two undetermined parameters in the system. The simulated control system transfer function was of the form shown in formula 3.3 The model transfer function as of the form shown in formula 3.4. In both cases the actual computer arrangement was that of the transfer function format. The system values of ω_n and zeta were set up through switching circuits so that three different values of each could be used directly. The adaptive loops used on the computer were those shown in Fig. 6 In general, both filters in the adaptive loops were set for the break at omega = .1 radian. Fig. 15 presents the full analog computer diagram for a simulated third order system with a model tracking two unknown parameters. This diagram conforms to the format of formula 3.3 and 3.5. The initial investigation with this computer arrangement concerned a control system with simulated open loop transfer function of:

$$G(s) = \frac{10\ (100)\ K}{s\ (s + p)\ (s + 100)} \qquad \begin{matrix} 5 \le K \le 15 \\ 5 \le p \le 15 \end{matrix} \qquad 4.1$$

and a model transfer function:

$$G(s) = \frac{1000\ (10 - m_c)}{s\ (s + 100)\ (s + 10 + m_c)} \qquad 4.2$$

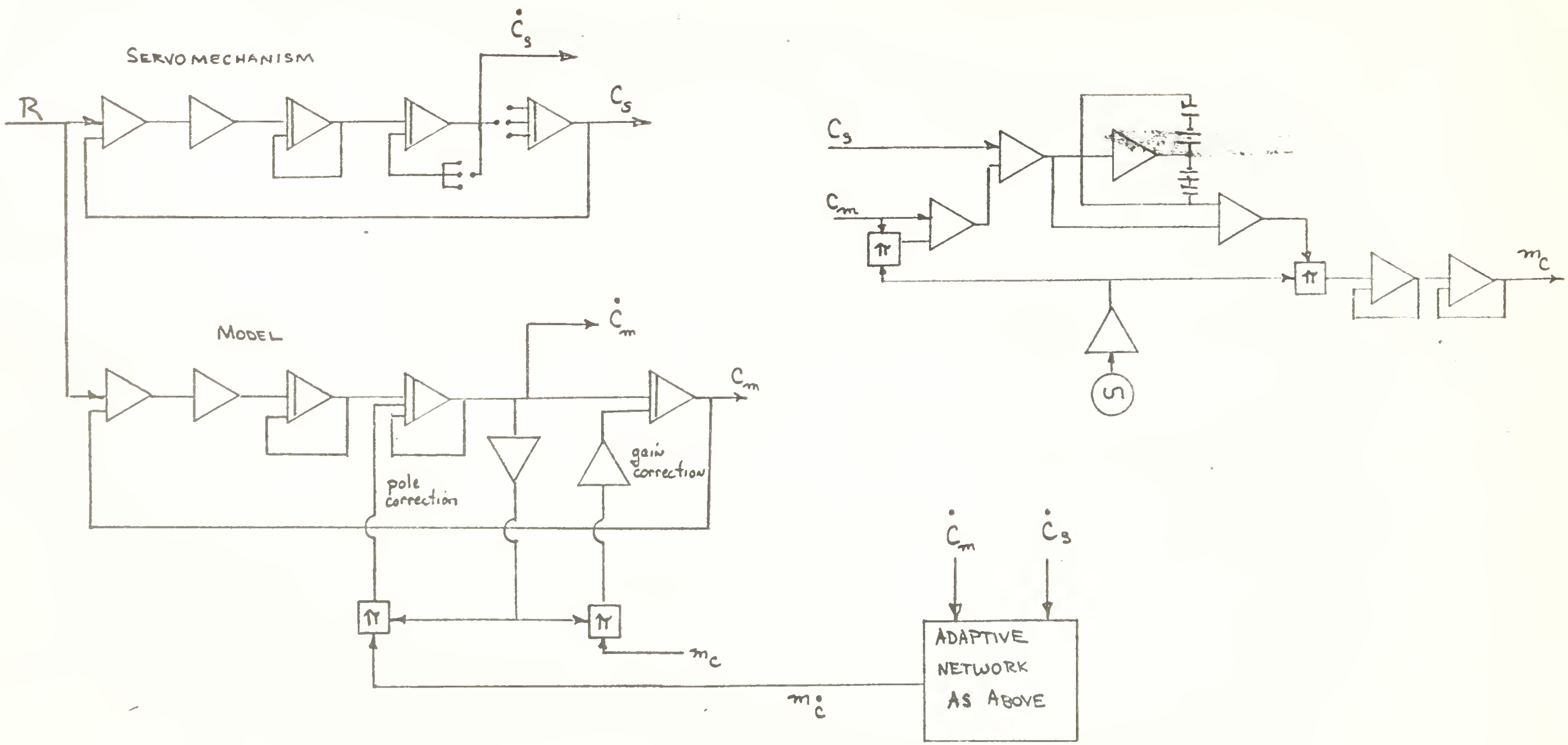


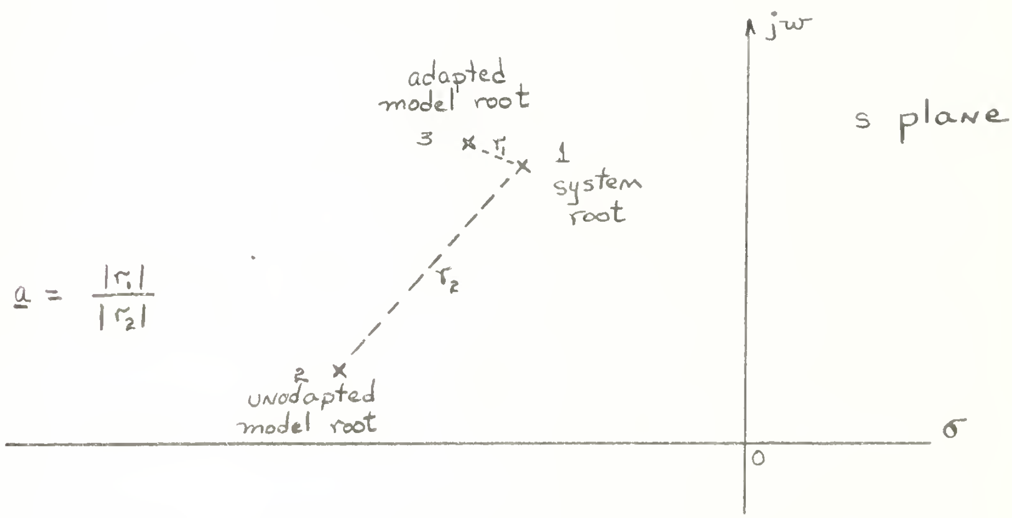
Figure 15

Analog Computer Diagram for Two Parameter Model Adaptation

The input was primarily deterministic. It consisted of a square wave with a frequency which was less than the bandwidth. Tests were also carried out however, using a random input. This random input was the output of a Gaussian "white" noise generator with an 18 db cut off at approximately the system bandwidth. At the completion of each run, the recorded m_c and $m_{\dot{c}}$ were used in conjunction with the known model characteristics to derive the closed loop characteristic equation of the simulated system. This allowed comparison of actual system zeta and ω_n with those derived from that identified as the system. A non-dimensional corrective parameter was examined and used as a partial criterion of identification. This corrective parameter $\underline{a}$ is best understood through examination of Fig. 16.

Fig. 16

CORRECTIVE PARAMETER



In this figure, (1) is the dominant root location of the simulated control system, (2) is the root location of the uncorrected model, and (3) is the position of the corrected model root. The arithmetic mean $\bar{\underline{a}}$ and the standard deviation, σ of $\underline{a}$, were computed for each series of tests.

The data for the first series of tests is shown in Table I. In this table the variance of the tracked parameters is shown. The roots, zeta and ω_n have been shown for only the two extreme cases in each test, that of the maximum and minimum zeta. The correction parameter a and the sigma have been calculated from the assembled data of the eight situations for each test where the parameters do not match. Fig. 17 shows a sample of the analog computer output. This data is that of the first and last cases tabulated in Table I Figure 17D will be used to illustrate the method of computing the characteristic equation roots of the operating control system. For this example, the model closed loop transfer function was

$$\frac{C}{R}(s)_m = \frac{10.7\,(s+14)\,(10-m_c)}{s\,(s+15)\,(s+10+m_{\dot{c}}) + 10.7\,(s+14)\,(10-m_c)} \qquad 4.3$$

From figure 17D the values of the correction factors may be seen to be $m_c = 1.5$ and $m_{\dot{c}} = 1.080$. The characteristic equation is then

$$s^3 + 26.08\,s^2 + 289.1\,s + 1724. \qquad 4.4$$

The roots of this equation were obtained through digital computer service. These roots are:

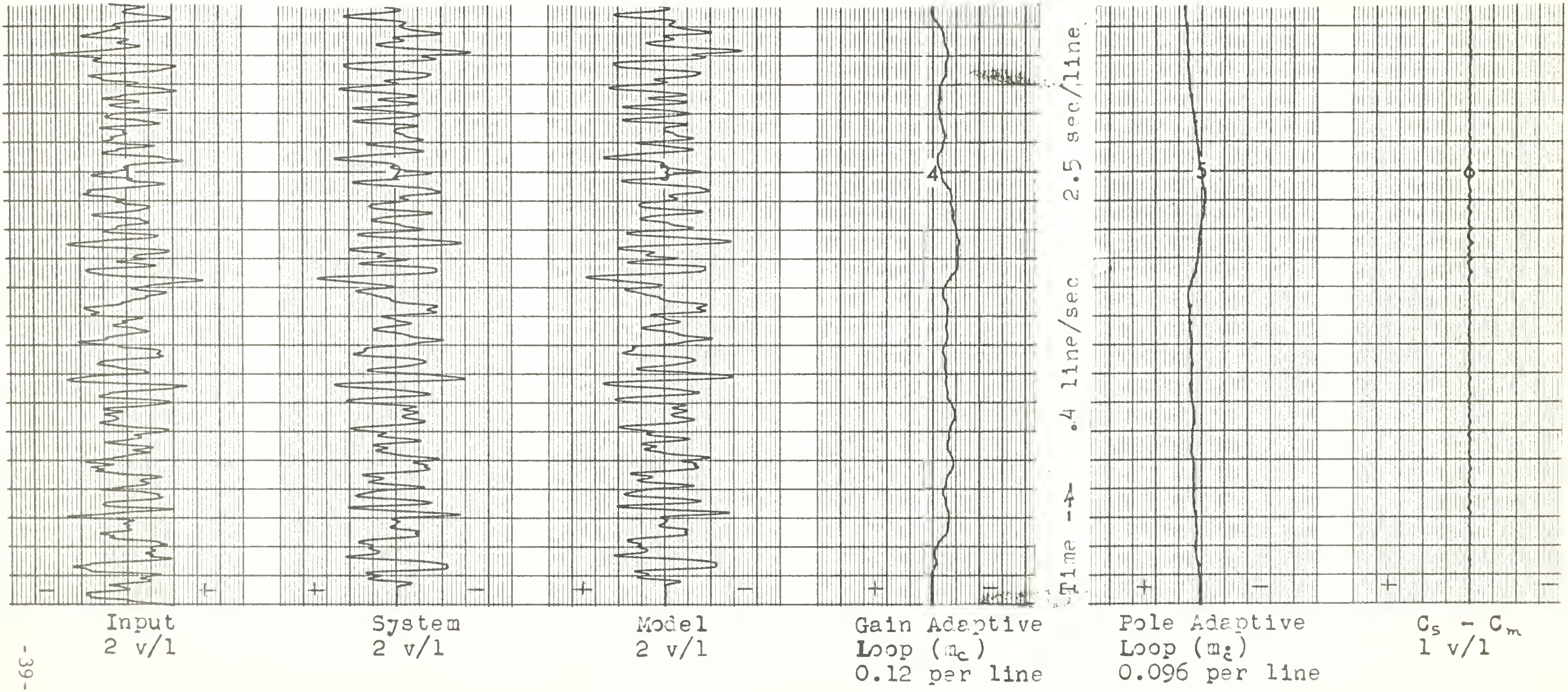
$$\begin{array}{l} -5.894 \pm j\,9.268 \\ -14.29 \end{array}$$

Fig. 18 shows a plot of the characteristic equation dominant root positions for the actual system and the identified system detailed in Fig. 17.

Equation Order Approximated

During the process of analyzing an operating system, it is inevitable that approximations would be made. These approximations will generally mask or ignore the very small time constants, which in the s plane are the poles of large magnitude. In trying to use the model modulation method to identify an operating system it is then inevitable that the model and the system would be of different order. For a step input, the effect of these poles would be greatest during the initial portion of the transient response, and the effect would become inconsequential in the later portions of the response. It was hypothesized that if the uncertain parameters of the operating system concerned dominant roots, satisfactory identification would take place. The restriction to this hypothesis was two fold. It should be expected that if the input was of a random noise nature with no stead state, no identification would be possible unless, as in the first test of Table I, the equation order and the additional parameters are known. Additionally, with deterministic inputs, identification could take place only when these unknown poles were of sufficient magnitude to be unimportant with respect to the dominant response.

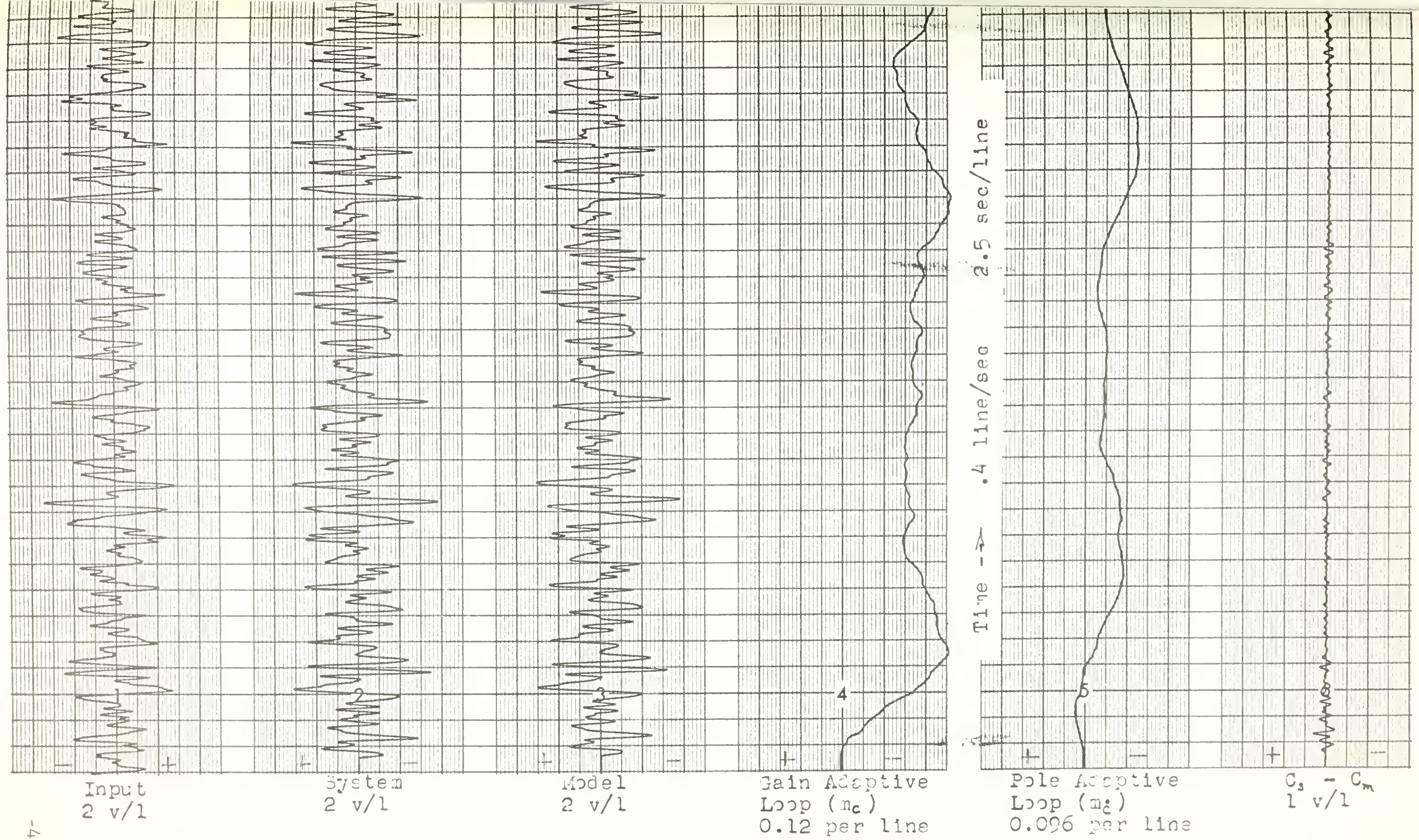
To examine this restriction the tests detailed in Table II were hold. Since the systems to be tested were set up in transfer function format on the analog computer, each pole was a new block set into the basic arrangement of Fig. 14. The information in Table II is of the same nature as in Table I, but as all nine system configurations differed from those of the model, all nine situations were used to calculate the $\underline{a}$ and σ.



Model $G(s) = \dfrac{1000(10 - m_c)}{s(s + 100)(s + 10 + m_\xi)}$ System $= \dfrac{10000}{s(s + 100)(s + 10)}$

Figure 17 a

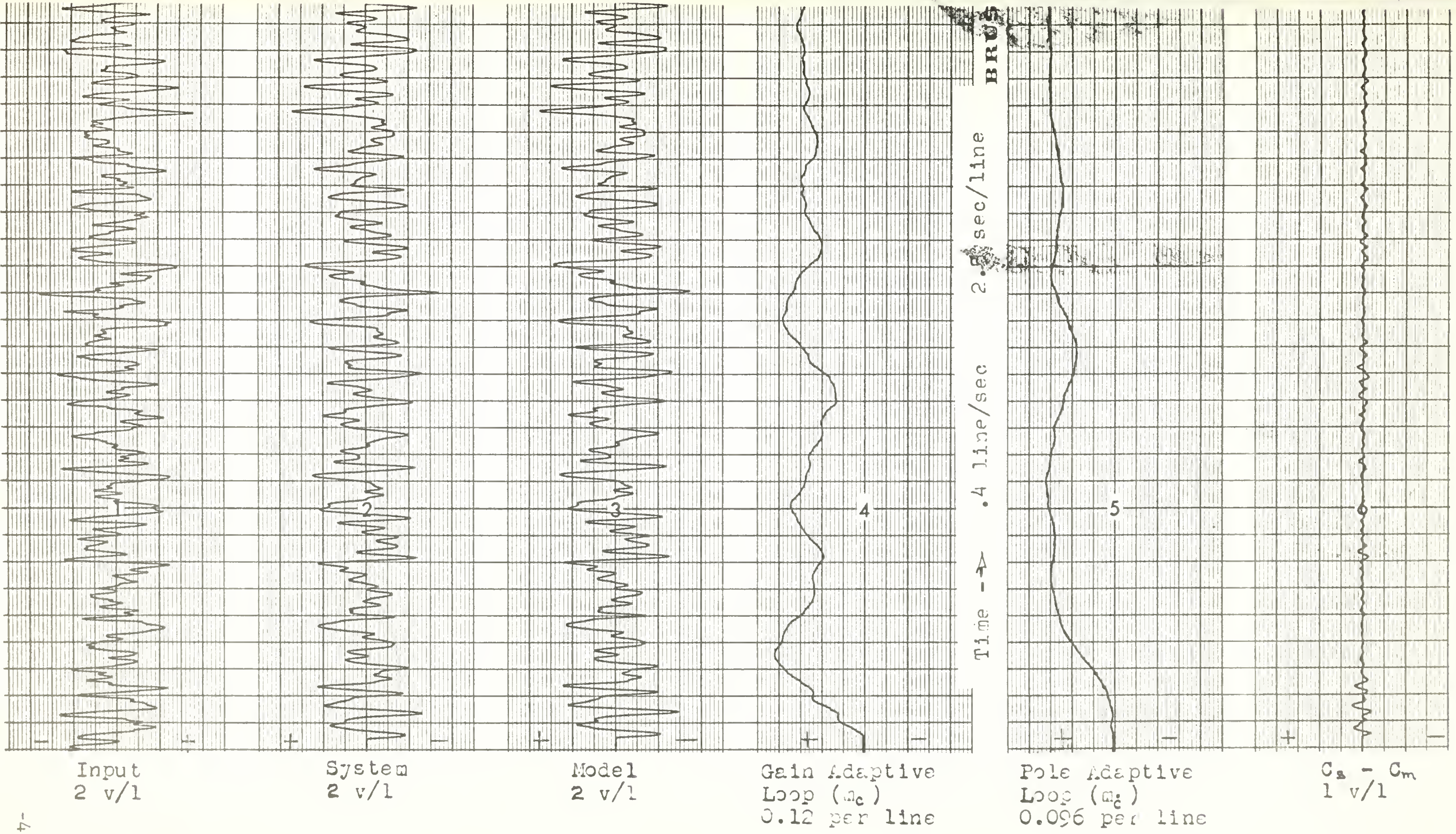
System Identification, Two Uncertain Parameters



$$\text{Model } G(s) = \frac{1000(10 - m_c)}{s(s + 100)(s + 10 + m_c')} \qquad \text{System } G(s) = \frac{12000}{s(s + 100)(s + 8)}$$

Figure 17b

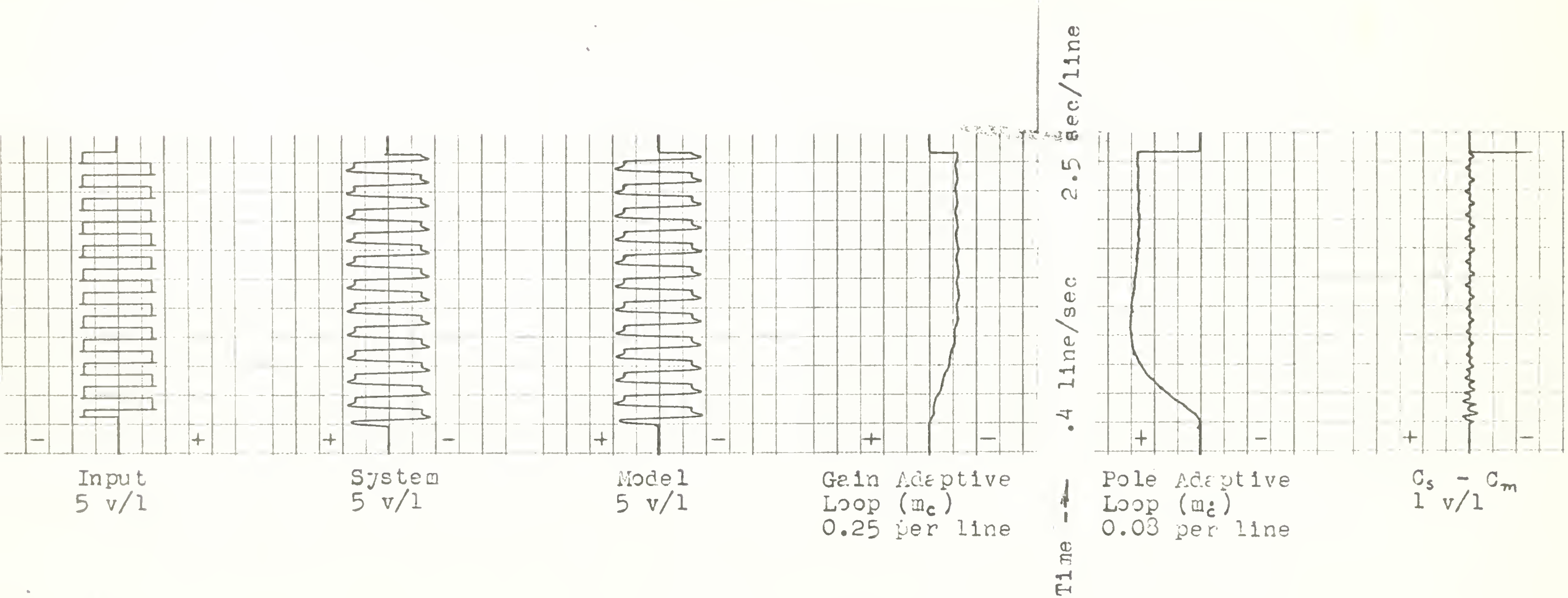
System Identification, Two Uncertain Parameters



$$\text{Model } G(s) = \frac{1000(10 - m_c)}{s(s + 100)(s + 10 + m_c')} \qquad \text{System } G(s) = \frac{8000}{s(s + 100)(s + 12)}$$

Figure 17c

System Identification, Two Uncertain Parameters

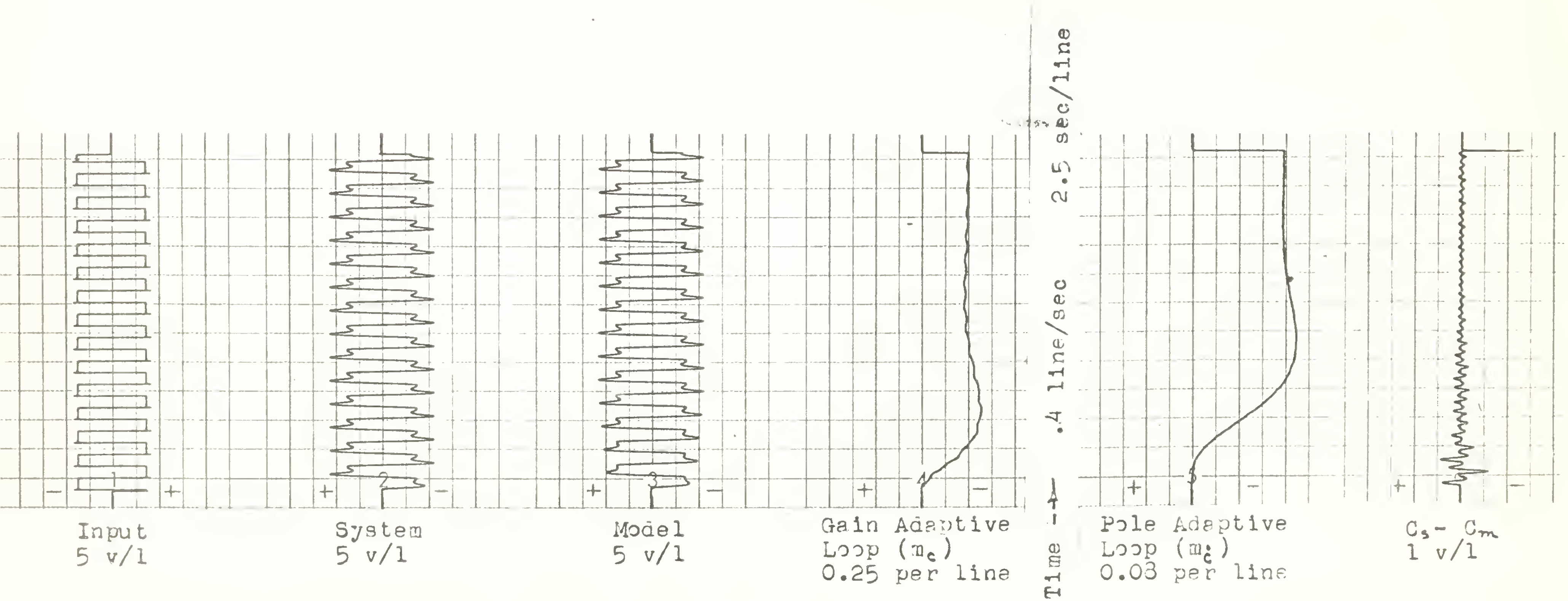


$$\text{System } G(s) = \frac{128.4(s + 14)}{s(s + 15)(s + 12)}$$

$$\text{Model } G(s) = \frac{10.7(s + 14)(10 - m_c)}{s(s + 15)(s + 10 + m_c')}$$

Figure 17d

System Identification, two Uncertain Parameters

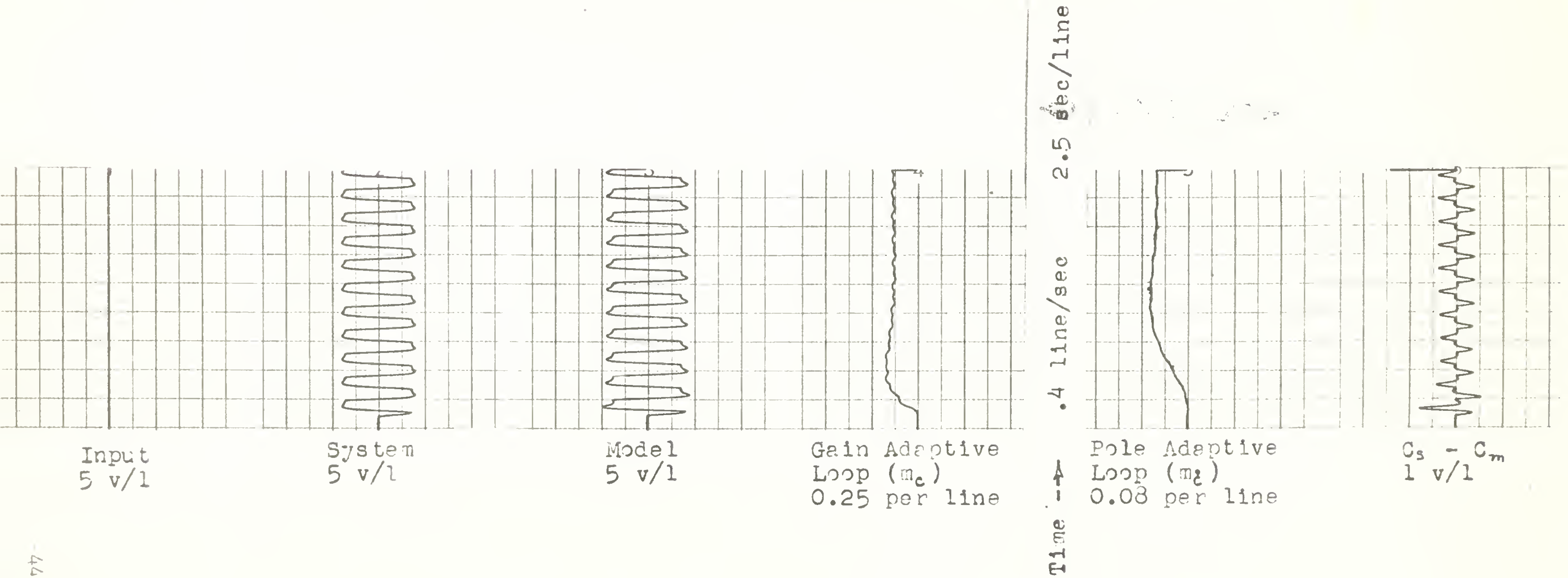


$$\text{System } G(s) = \frac{128.4(s + 14)}{s(s + 15)(s + 3)}$$

$$\text{Model } G(s) = \frac{10.7(s + 14)(10 - m_c)}{s(s + 15)(s + 10 + m_c')}$$

Figure 17e

System Identification, Two Uncertain Parameters



$$\text{System } G(s) = \frac{35.6(s + 14)}{s(s + 15)(s + 12)}$$

$$\textbf{Model } G(s) = \frac{10.7(s + 14)(10 - m_c)}{s(s + 15)(s + 10 + m_\ell)}$$

Figure 17f

System Identification, Two Uncertain Parameters

Table I

SYSTEM	MODEL	ACTUAL SYSTEM			IDENTIFIED			$\bar{a}$	σ
		ROOTS	ZETA	ω_n	ROOTS	ZETA	ω_n		
$\frac{10000}{s(s+100)(s+10)}$	$\frac{10000}{s(s+100)(s+10)}$	$-4.45\pm j8.89$	.45	9.9	$-4.41\pm j8.92$	.45	10.0		
$\frac{12000}{s(s+100)(s+8)}$	"	$-3.36\pm j10.35$	.31	10.9	$-3.97\pm j10.10$	.36	10.9	.370	.1225
$\frac{8000}{s(s+100)(s+12)}$	"	$-5.56\pm j6.96$	.62	8.9	$-5.26\pm j7.48$	.59	9.1		
$\frac{10000}{s(s+100)(s+10)}$	$\frac{10000}{s(s+100)(s+10)}$	$-4.45\pm j8.89$	.45	9.9	$-4.47\pm j8.85$	.45	9.9		
$\frac{12000}{s(s+100)(s+8)}$	"	$-3.36\pm j10.35$	.31	10.9	$-3.56\pm j10.13$	.33	10.8	.216	.0977
$\frac{8000}{s(s+100)(s+12)}$	"	$-5.56\pm j6.96$	.62	8.9	$-5.42\pm j7.03$	.62	8.9		
$\frac{3000}{s(s+30)(s+10)}$	$\frac{3000}{s(s+30)(s+10)}$	$-3.12\pm j8.89$	.33	9.45	$-3.15\pm j8.93$	.33	9.3		
$\frac{3600}{s(s+30)(s+8)}$	"	$-1.97\pm j10.09$	.19	10.3	$-2.16\pm j10.10$	.21	10.3	.221	.0912
$\frac{2400}{s(s+30)(s+12)}$	"	$-4\ 31\pm j7.30$	.51	8.5	$-3.99\pm j7.51$	.47	8.55		

Table I

SYSTEM	MODEL	ACTUAL SYSTEM			IDENTIFIED			$\bar{a}$	σ
		ROOTS	ZETA	ω_n	ROOTS	ZETA	ω_n		
$\frac{10000}{s(s+100)(s+10)}$	$\frac{10000}{s(s+100)(s+100)}$	$-4.45\pm j8.85$	.46	9.9	$-4.25\pm j8.87$	.44	9.85		
$\frac{15000}{s(s+100)(s+5)}$	"	$-1.72\pm j12.05$	.15	12.2	$-3.02\pm j11.5$	.25	12.05	.427	.0120
$\frac{5000}{s(s+100)(s+15)}$	"	$-7.2\pm j1.5$	.98	7.4	$-8.05\pm j4.85$	.85	9.4		
$\frac{10000}{s(s+100)(s+10)}$	$\frac{10000}{s(s+100)(s+10)}$	$-4.45\pm j8.87$	.45	10.0	$-4.45\pm j8.87$	.45	10.6		
$\frac{12000}{s(s+100)(s+5)}$	"	$-1.85\pm j10.75$	.18	10.9	$-1.68\pm j11.45$	.15	11.6	.199	.0735
$\frac{8000}{s(s+100)(s+15)}$	"	$-7.02\pm j5.45$	.79	8.9	$-6.34\pm j5.40$	.77	8.4		
$\frac{10000}{s(s+50)(s+10)}$	$\frac{10000}{s(s+50)(s+10)}$	$-2.95\pm j13.25$	.23	13.6	$-2.92\pm j13.11$	.23	13.5		
$\frac{12000}{s(s+50)(s+10)}$	"	$-1.65\pm j14.67$	.12	14.8	$-1.70\pm j14.82$	.12	14.9	.152	.0840
$\frac{8000}{s(s+50)(s+12)}$	"	$-4.20\pm j11.45$	.355	12.2	$-4.06\pm j11.21$	.35	12.0		

Table I

SYSTEM	MODEL	ACTUAL SYSTEM			IDENTIFIED			$\bar{a}$	σ
		ROOTS	ZETA	ω_n	ROOTS	ZETA	ω_n		
$\frac{2500}{s(s+25)(s+10)}$	$\frac{2500}{s(s+25)(s+10)}$	-2.80 ± 8.78	.30	9.3	$-2.83\pm j8.78$	.3	9.3		
$\frac{3000}{s(s+25)(s+8)}$	"	$-1.66\pm j9.91$	.17	10.1	$-1.84\pm j9.93$	.18	10.1	.259	.1072
$\frac{2000}{s(s+20)(s+12)}$	"	$-3.97\pm j7.28$	.48	8.3	$-3.73\pm j7.46$	.46	8.35		
$\frac{2000}{s(s+20)(s+10)}$	$\frac{2000}{s(s+20)(s+10)}$	$-2.39\pm j8.57$	.26	8.9	$-2.50\pm j8.62$	.28	9.0		
$\frac{2400}{s(s+20)(s+10)}$	"	$-2.10\pm j9.45$	.22	9.65	$-2.20\pm j9.34$	.23	9.6	.212	.0945
$\frac{1600}{s(s+20)(s+10)}$	"	$-3.52\pm j7.19$	.44	8.0	$-3.33\pm j7.45$	.42	8.2		
$\frac{128.4(s+14)}{s(s+15)(s+12)}$	$\frac{107(s+14)}{s(s+15)(s+10)}$	$-6.40\pm j9.25$	.54	11.0	$-5.89\pm j9.26$	.57	11.3		
$\frac{128.4(s+14)}{s(s+15)(s+8)}$	"	$-4.28\pm j10.30$	.39	11.2	$-4.52\pm j10.46$	.40	11.4	.270	.1314
$\frac{85.6(s+14)}{s(s+15)(s+12)}$	"	$-6.36\pm j6.57$	.70	9.3	$-5.59\pm j7.72$	.67	10.1		

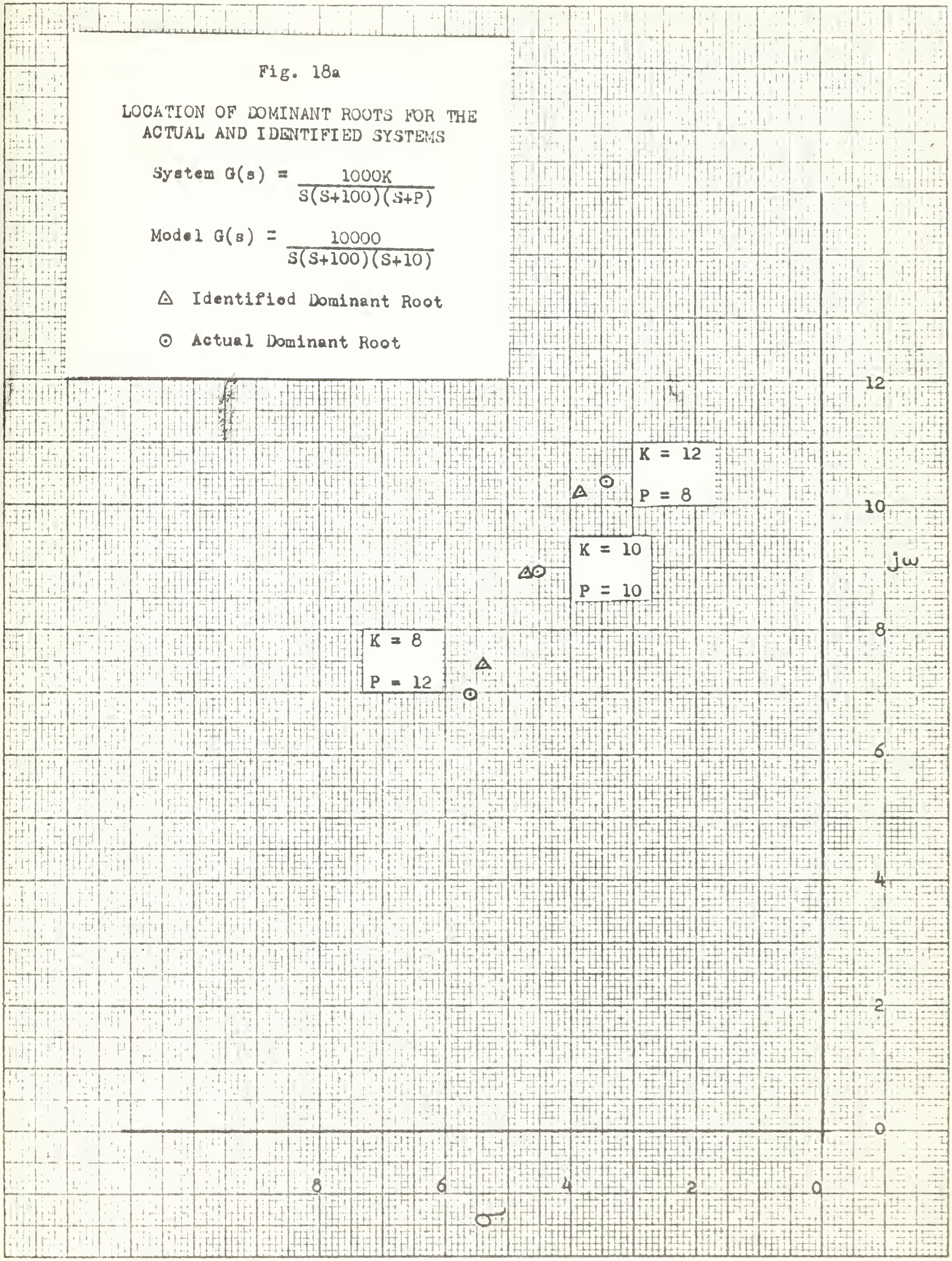


Fig. 18a

LOCATION OF DOMINANT ROOTS FOR THE ACTUAL AND IDENTIFIED SYSTEMS

System $G(s) = \dfrac{1000K}{S(S+100)(S+P)}$

Model $G(s) = \dfrac{10000}{S(S+100)(S+10)}$

△ Identified Dominant Root

⊙ Actual Dominant Root

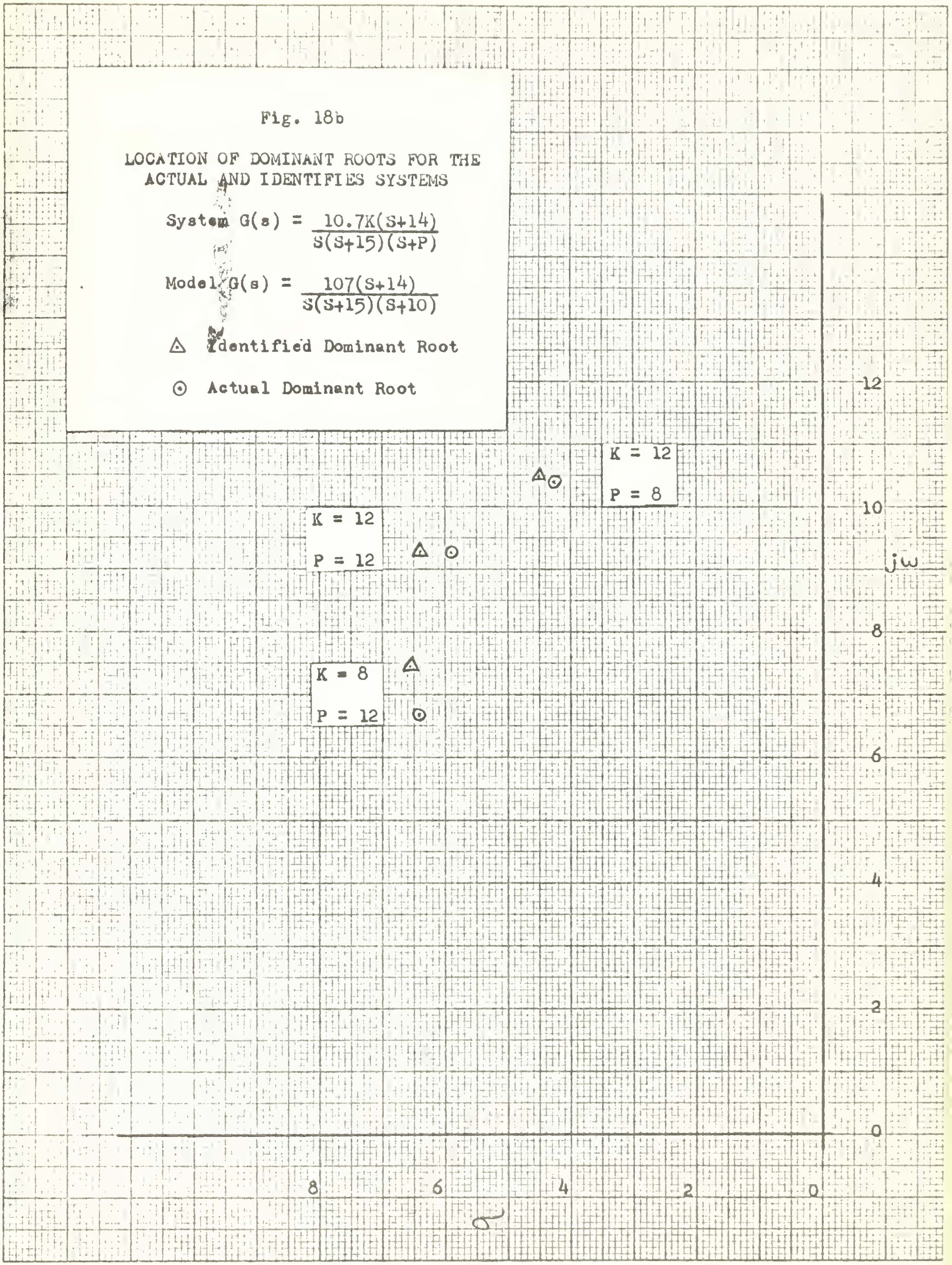


Fig. 18b

LOCATION OF DOMINANT ROOTS FOR THE ACTUAL AND IDENTIFIES SYSTEMS

$$\text{System } G(s) = \frac{10.7K(S+14)}{S(S+15)(S+P)}$$

$$\text{Model } G(s) = \frac{107(S+14)}{S(S+15)(S+10)}$$

△ Identified Dominant Root

⊙ Actual Dominant Root

The results of these tests were as expected. The random input revealed, as expected, lack of identification. Under deterministic inputs, when the postulated restrictions were held, satisfactory identification took place. One interesting point revealed in this investigation was that an adequate test for satisfactory identification would be made if the frequency of the input could be varied. Naturally this violates the basic restriction that led to the development of this method, i.e., no violation of the natural operation of the system, but is a point of interest. For this process the frequency was varied between a lower frequency with a period equal to the second order time constant and an upper frequency equal to the bandwidth. When the adaptive loop outputs remained relatively constant through this frequency range it was found that these outputs served to adequately identify the dominant roots when the characteristic roots were found. If, however, the identification varied through this range, the effect of the ignored poles precluded satisfactory identification.

Figure 19 presents illustrative data from the first tablulated case of Table II for this series of tests. Fig. 20 shows the dominant roots of the system for each of the tests illustrated in Fig. 19, with the dominant roots as identified from the data.

Examination of the data reveals that for both of these types of equation situations good dominant root identification has been achieved. It was found that these dominant roots could be identified to within an average error of 2.2% of the undamped natural frequency and to within an average error of 4.76% of the complex phase angle. When

this method is used to identify the characteristics of an actual linear control system, it is highly probable that greater accuracy may be attained since the model parameters may be varied to obtain error bracketing. This process is illustrated in Chapter VI.

Table II

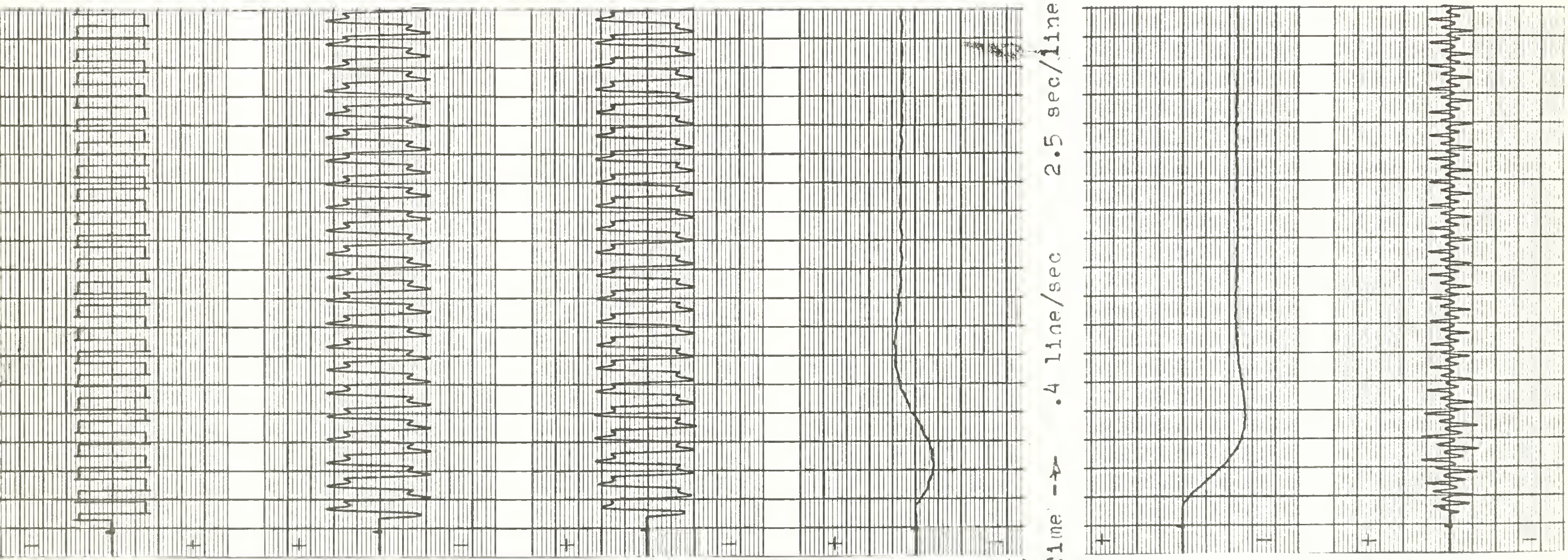
SYSTEM	MODEL	ACTUAL SYSTEM			IDENTIFIED			$\bar{a}$	σ
		ROOTS	ZETA	ω_n	ROOTS	ZETA	ω_n		
$\frac{10000}{s(s+100)(s+10)}$	$\frac{100}{s(s+10)}$	$-4.45\pm j8.89$	.45	9.9	$-4.67\pm j8.75$	.48	10.0		
$\frac{12000}{s(s+100)(s+8)}$	"	$-3.36\pm j10.35$	.31	10.9	$-3.75\pm j10\ 1$	35	10.9	.202	0858
$\frac{8000}{s(s+100)(s+12)}$	"	$-5.56\pm j6.96$	.62	8.9	$-5.60\pm j6\ 90$	.63	8.9		
$\frac{10000}{s(s+100)(s+10)}$	$\frac{100}{s(s+10)}$	$-4.45\pm j8.89$	.45	9.9	$-4.69\pm j8.90$	.46	10.1		
$\frac{12000}{s(s+100)(s+8)}$	"	$-3.36\pm j10.35$	.31	10.9	$-4.00\pm j10.4$	.36	11.2	.263	.0732
$\frac{8000}{s(s+100)(s+12)}$	"	$-5.56\pm j6.96$	.62	8.9	$-5.40\pm j7.30$	.61	9.1		
$\frac{10000}{s(s+100)(s+10)}$	$\frac{100}{s(s+10)}$	$-4.45\pm j8.89$	.45	9.9	$-4.95\pm j8.70$	.50	10.1		
$\frac{12000}{s(s+100)(s+8)}$	"	$-3\ 78\pm j10.10$	.31	10.9	$-3.78\pm j10.10$	.36	10.8	.332	.1540
$\frac{8000}{s(s+100)(s+12)}$	"	$-5.56\pm j6\ 96$	.62	8.9	$-5.54\pm j7\ 62$	.60	9 5		

Table II

SYSTEM	MODEL	ACTUAL SYSTEM			IDENTIFIED			$\bar{a}$	σ
		ROOTS	ZETA	ω_n	ROOTS	ZETA	ω_n		
$\frac{1,000,000}{s(s+100)(s+100)(s+10)}$	$\frac{100}{s(s+10)}$	$-3.89\pm j9.04$	.40	9.9	$-4.23\pm j9.38$	.41	10.3		
$\frac{1,200,000}{s(s+100)(s+100)(s+8)}$	"	$-2.72\pm j10.40$	.26	10.8	$-3.42\pm j11.40$	.28	11.9	.336	.0485
$\frac{800,000}{s(s+100)(s+100)(s+12)}$	"	$-5.07\pm j7.22$	.58	8.9	$-5.00\pm j7.56$	.55	9.2		
$\frac{7500}{s(s+75)(s+10)}$	$\frac{100}{s(s+10)}$	$-4.26\pm j8.93$	.44	9.9	$-4.58\pm j9.30$	.45	10.4		
$\frac{9000}{s(s+75)(s+8)}$	"	$-3.14\pm j10.36$	.31	10.9	$-3.80\pm j10.50$	.35	11.2	.332	.1540
$\frac{6000}{s(s+75)(s+8)}$	"	$-5.38\pm j7.04$	.61	8.9	$-5.36\pm j7.15$	.61	9.0		
$\frac{10000}{s(s+100)(s+10)}$	$\frac{100}{s(s+10)}$	$4.45\pm j8.89$	.45	8.9	$-4.82\pm j8.90$	.47	10.1		
$\frac{15000}{s(s+100)(s+5)}$	"	$-3.36\pm j10.35$	.14	13.2	$-3.20\pm j11.20$	.27	11.7	.387	.1460
$\frac{5000}{s(s+100)(s+15)}$	"	$-7.20\pm j1.5$	1.0	7.2	$-6.63\pm j1.6$	.97	6.9		

Table II

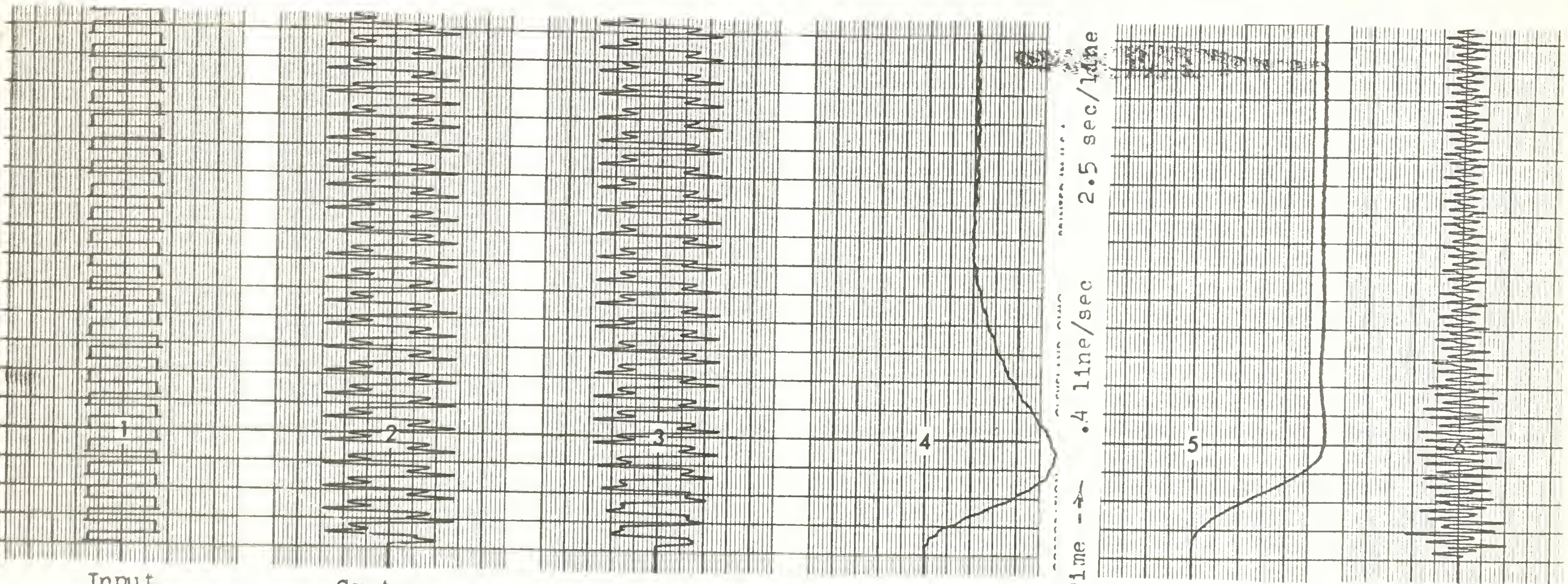
SYSTEM	MODEL	ACTUAL SYSTEM			IDENTIFIED			$\bar{a}$	σ
		ROOTS	ZETA	ω_n	ROOTS	ZETA	ω_n		
$\frac{750,000}{s(s+75)(s+100)(s+10)}$	$\frac{100}{s(s+10)}$	$-3.69\pm j9.05$	.38	9.8	$-4.24\pm j8.85$	.43	9.9		
$\frac{900,000}{s(s+75)(s+100)(s+8)}$	"	$-2.51\pm j10.37$	.25	10.7	$-3.18\pm j10.15$	.30	10.7	.304	.1268
$\frac{600,000}{s(s+75)(s+100)(s+12)}$	"	$-4.90\pm j7.28$	.57	8.8	$-5.07\pm j7.30$	.58	8.9		
$\frac{562,200}{s(s+75)(s+75)(s+10)}$	$\frac{100}{s(s+10)}$	$-3.51\pm j9.06$	.37	9.8	$-4.14\pm j8.90$	.43	9.8		
$\frac{774,640}{s(s+75)(s+75)(s+8)}$	"	$-2.31\pm j10.34$	.24	10.6	$-3.02\pm j10.20$	.29	10.5	.332	.1148
$\frac{449,760}{s(s+75)(s+75)(s+12)}$	"	$-4.73\pm j7.33$	.55	8.8	$-5.10\pm j7.38$	.57	9.0		
$\frac{250,000}{s(s+50)(s+50)(s+10)}$	$\frac{100}{s(s+10)}$	$-3.97\pm j9.21$	.26	11.0	$-3.88\pm j9.23$	.28	12.1		
$\frac{300,000}{s(s+50)(s+50)(s+8)}$	"	$-2.74\pm j10.62$	.40	10.1	$-3.35\pm j11.60$	.39	10.1	.210	.1138
$\frac{200,000}{s(s+50)(s+50)(s+12)}$	"	$-5.20\pm j7.31$	.58	9.1	$-5.14\pm j7.42$	.57	9.1		



Model $G(s) = \frac{10(10 - m_c)}{s(s + 10 + m_{\dot{c}})}$ System $G(s) = \frac{750{,}000}{s(s + 75)(s + 100)(s + 10)}$

Figure 19 a

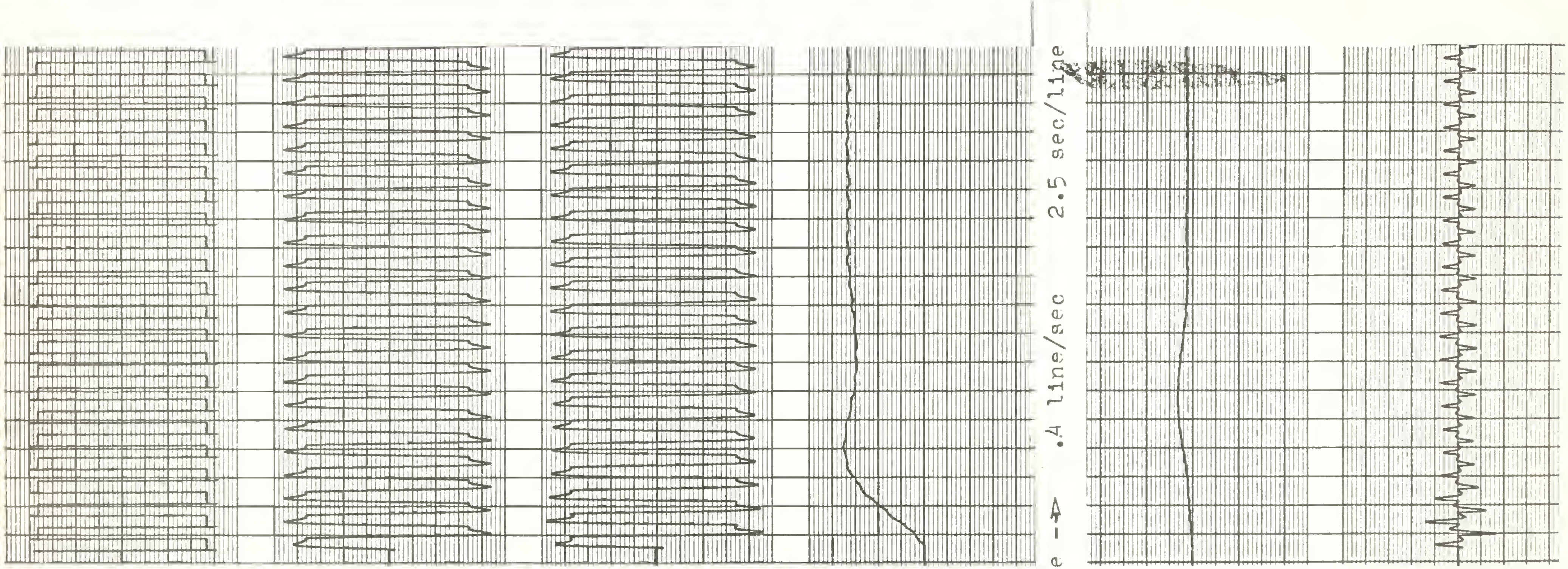
System Dominant Root Identification, Two Uncertain Parameters



$$\text{Model } G(s) = \frac{10(10 - m_c)}{s(s + 10 + m_\xi)} \qquad \text{System } G(s) = \frac{900{,}000}{s(s + 75)(s + 100)(s + 10)}$$

Figure 19 b

System Dominant Root Identification, Two Uncertain Parameters



$$\text{Model } G(s) = \frac{10(10 - m_c)}{s(s + 10 + m_c)} \qquad \text{System } G(s) = \frac{600{,}000}{s(s + 75)(s + 100)(s + 12)}$$

Figure 19c

System Dominant Root Identification, Two Uncertain Parameters

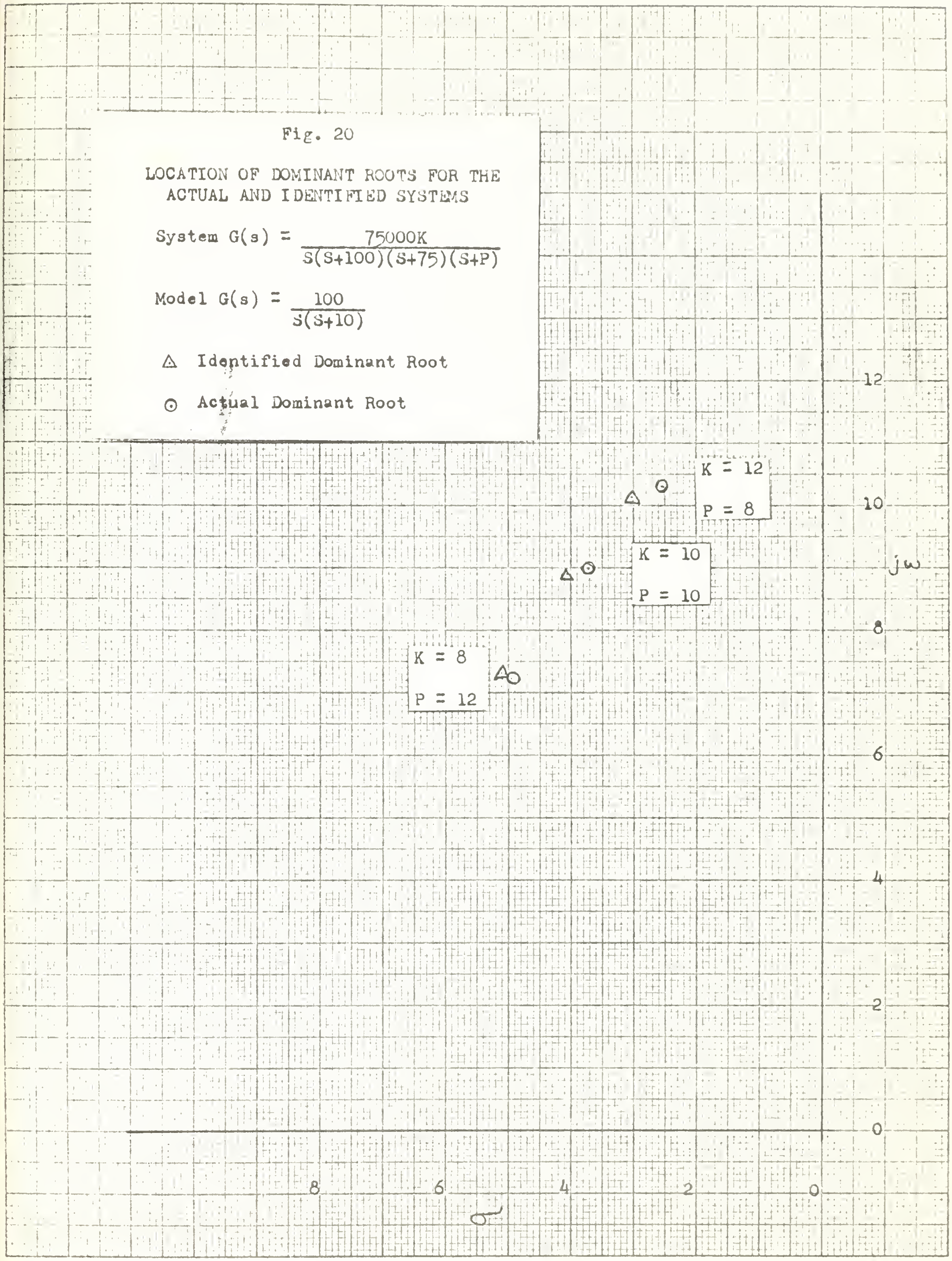


Fig. 20

LOCATION OF DOMINANT ROOTS FOR THE ACTUAL AND IDENTIFIED SYSTEMS

System $G(s) = \dfrac{75000K}{S(S+100)(S+75)(S+P)}$

Model $G(s) = \dfrac{100}{S(S+10)}$

△ Identified Dominant Root

⊙ Actual Dominant Root

Chapter V

DIGITAL COMPUTER SIMULATION

Direct utilization of a digital computer for identification of an operating control system was not considered It is possible, however, that operating systems might be monitored or controlled by digital techniques. In this case the output would be presented in the form of discrete numerical data. Ideally this data could be used as input to a digital-to-analog converter, and the analog techniques previously described could be used for operating system identification. In the absence of this equipment, digital simulation of the control system, model, and adaptive loops was investigated to illustrate the techniques and the potential applicability of model modulation techniques through numerical techniques. This investigation was conducted through the use of the CDC 1604 Digital Computer. The program code was written in Fortran. A standard Runge-Kutta numerical integration process was used to solve the differential equation which governed the operation of the full model, system and adaptive loop. The differential equations were formulated in a transfer function format.

Three basic configurations of model adaptation were tested through this digital computer simulation. For these configurations the model was used to identify the operating control system characteristic equation roots under the following conditions:

1. Second order control system with undetermined pole.
2. Second order model, third order control system with uncertain gain and uncertain dominant pole.
3. Third order system and model with two uncertain poles.

The actual investigation was limited to achieving a successful solution for one parameter set for each of these configurations. The excessive time used by integration techniques on the digital computer was of course a limiting factor. The first of these configurations will be by-passed in this discussion since it can be considered to be a subsystem of the second configuration.

The model for the second configuration had the following open loop transfer function:

$$G(s) = \frac{10\,(10 - m_c)}{s\,(s + 10 + \dot{m}_c)} \qquad 5.1$$

and a closed loop transfer function of

$$\frac{C}{R}(s)_m = \frac{10\,(10 - m_c)}{s\,(s + 10 + \dot{m}_c) + 10\,(10 - m_c)} \qquad 5.2$$

The control system open loop transfer functions were

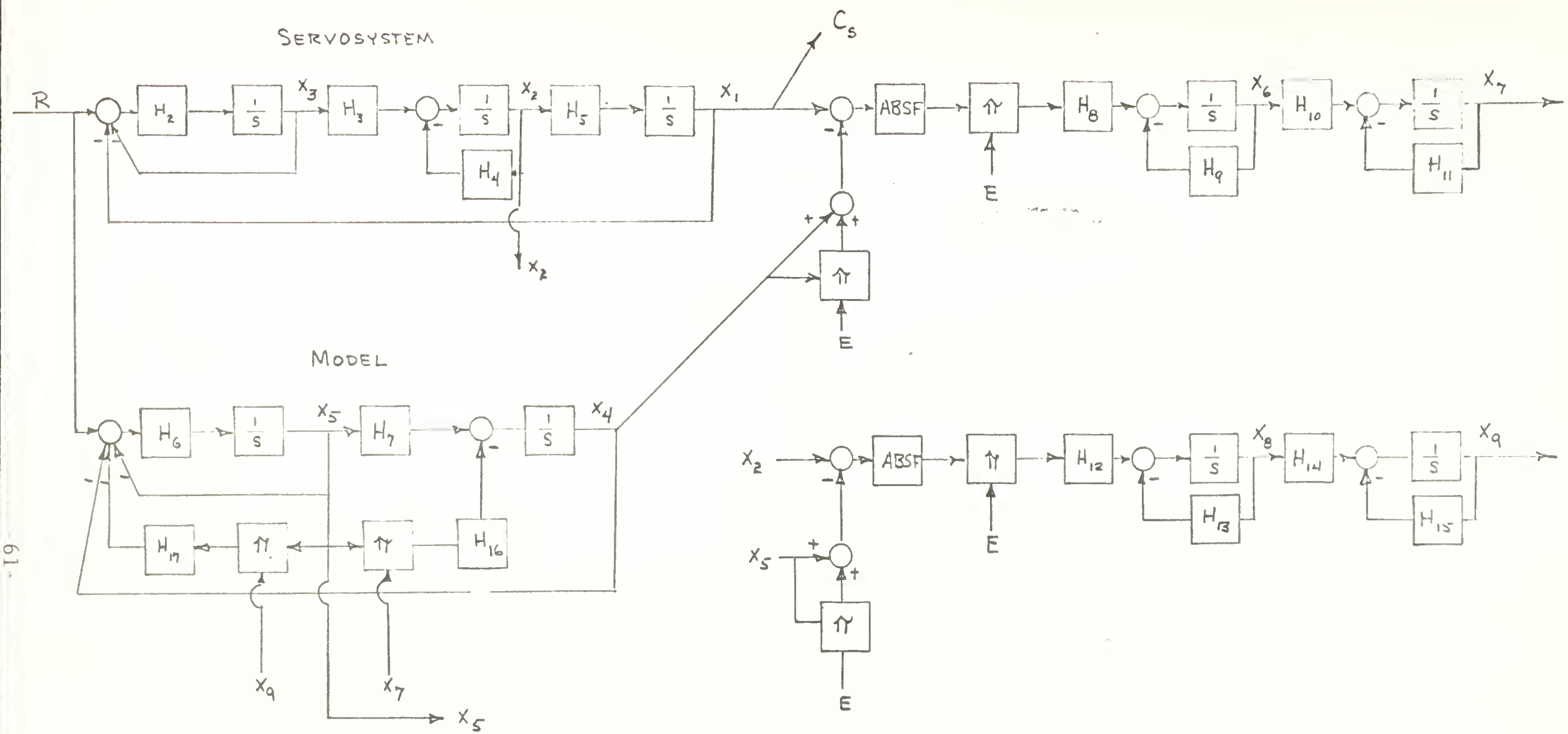
$$G(s) = \frac{12000}{s(s+100)(s+8)} \qquad 5.3$$

and

$$G(s) = \frac{8000}{s(s+100)(s+12)} \qquad 5.4$$

Figure 21 presents the transfer function format which was used to formulate the differential equation for the FORTRAN program. This format allowed formulation of the operation by a set of quasi-state variables of the form:

$$\dot{x}_n = f_n\,(x_1, x_2, x_3, 000\; x_n, R) \qquad 5.5$$




```
2 E = SINF(314.*T)
  H(4) = VARIABLE PCLE
  H(5) = VARIABLE GAIN
  XDOT(1) = H(5)*X(2)
  XDOT(2) = X(3)*H(3) - X(2)*H(4)
  XDOT(3) = H(2)*(R - X(1)- X(3))
  XDOT(4) = X(5)*(H(7) - X(7)*H(16))
  XDOT(5) = H(6)*(R - X(5) - X(5)*X(9)*H(17) - X(4))
  XDOT(6) = H(8)*E*ABSF(X(1)- X(4) - E*X(4)) - X(6)*H(9)
  XDOT(7) = H(10)*X(6) - X(7)*H(11)
  XDOT(8) = H(12)*E*ABSF( X(2) - X(5) - X(5)*E) - X(8)*H(13)
  XDOT(9) = H(14)*X(8) - H(15)*X(9)
```

Figure 21

Transfer Function Format for Digital Computer Program

The solution of the equations was programmed to be presented in both numerical and graphical format. Fig. 22 presents a sample of the input program Fig. 23 presents a sample of the numerical data output, and Fig. 24 presents the graphical data output for the control system open loop transfer function given by equation 5 3.

The graph of Fig. 24 presents the corrective parameters as "corrected pole" = $(10 + m_c)$ and "corrected gain" = $(10 - m_c)$

Table III shows the identification of the characteristic equation roots for these two situations.

Table IIIA

DIGITAL COMPUTER CHARACTERISTIC EQUATION ROOTS

	Control System	Model as Corrected
Equation 5.3	$-3.36 \pm j\,10.35$ -101.27	$-3.07 \pm j\,9.97$ -101.15
Equation 5.4	$-5.55 \pm j\,6.96$ -100.9	$-4.88 \pm j\,7.77$ -100.93

Figure 22 a

Digital Computer Program

```
      PROGRAM ADAPT 12
C N=NO.OF EQNS., NP=NO. OF INCREMENTS BETWEEN PRINT OUTS, TO=INITIAL
C TIME, TF=FINAL TIME, DT=TIME INCREMENT, TN=TIME FIRST PRINTED DATA,
C X(J)=INITIAL VALUES OF THE VARIABLES X(J)
C NG=NO. OF INCREMENTS BETWEEN GRAPH POINTS
     0DIMENSION X(30),X1(900),X2(900),X3(900),X4(900),X5(900),X6(900),TI
     1ME(900), H(30)
      COMMON R, E, T2, H
  100 FORMAT (3I10/(8F10.0))
  102 FORMAT ( I2 / (I2,F20.0))
  200 FORMAT (F8.3,10F11.5)
  2010FORMAT (99H   TIME     SYS.POS.    MOD.POS.    POS.DIFF.   SYS.VEL.
     1MOD.VEL.    VEL.DIFF.  GAIN COR.  POLE COR.                         //)
  202 FORMAT (/)
  203 FORMAT (1H1)
  204 FORMAT (//)
  205 FORMAT (19H SYSTEM= F9.1, 13H/S(S+100)(S+ F6.2, 2H).   / )
  2060FORMAT (32H GAIN CORR FILTER CORNER AT W = F6.2  , 27H, POLE CORR.
     1 FILTER AT W = F6.2              /)
  207 FORMAT (25H MODEL = 100./S(S+10).          / )
  2080FORMAT (37H ADAPTIVE SYSTEM GAINS, OUTPUT (S) = F6.2, 20H,FIRST DE
     1RIV. (D) = F6.2               /)
  210 FORMAT (20H COEFFICIENT NUMBER I2,  2H =  F12.4, 1H.  )
      READ 100, N, NP, NG, TO, TF, DT, TN, (X(J), J=1,N)
      READ 102, NH, (  K, (H(K)), I = 1,NH )
      PRINT 204
      PRINT 208, H(16), H(17)
      ASYSG = H(5)* H(2)* H(3)
      PRINT 205 , ASYSG, H(4)
      PRINT 206 , H(11), H(15)
      PRINT 207
      DO 6  I = 1,NH
      PRINT 202
    6 PRINT 210, I, H(I)
      PRINT 203
      PRINT 201
      T=TO
      T2 = 0.0
      NUMPTS = 0
      MPTS= 0
    1 DPOS = X(1) - X(4)
      DVEL = X(2) - X(5)
      AGACOR = H(7) - X(7)*H(16)
      APOLCOR = H(6)*( 1. + X(9)*H(17))
      IF (T-TN) 36, 2, 2
    2 IF(MPTS)  30,35,30
   30 IF(XMODF(MPTS,50*NP)) 31,33,31
   31 IF(XMODF(MPTS,10*NP)) 32,34,32
   32 IF(XMODF(MPTS,    NP)) 36,35,36
   33 PRINT 203
      PRINT 201
   34 PRINT 202
   35 PRINT 200, T,X(1),X(4),DPOS,X(2),X(5), DVEL, AGACOR,APOLCOR
   36 MPTS=MPTS+1
   37 IF(XMODF(MPTS, NG)) 38, 39, 38
   39 NUMPTS = NUMPTS+1
      IF (900 - NUMPTS ) 38, 38, 40
   40 X1(NUMPTS) = X(1)
      X2(NUMPTS) = X(4)
      X3(NUMPTS) = AGACOR
      X4(NUMPTS) = APOLCOR
      X5(NUMPTS) = DPOS
      X6(NUMPTS) = DVEL
      TIME (NUMPTS) = T
   38 IF(TF-T-DT)  10,20,20
   10 PRINT 204
      CALL GRAPH ( NUMPTS , TIME , X3 , 8 )
      CALL GRAPH ( NUMPTS , TIME , X4 , 8 )
      CALL GRAPH ( NUMPTS , TIME , X5 , 8 )
      CALL GRAPH ( NUMPTS , TIME , X6 , 8 )
      CALL GRAPH ( NUMPTS , TIME , X1 , 8 )
      CALL GRAPH ( NUMPTS , TIME , X2 , 8 )
      STOP
   20 CALL RKUTTA(N, T, X, DT)
      T2= T2 + DT
      T = T + DT
      GO TO 1
```

Digital Computer Program
(con't)

```
      END
      SUBROUTINE RKUTTA (N,T,X,DT)
      DIMENSION  X(30),AK(4,30) , XDOT(30), XC(30),C(4) , H(30)
      COMMON R, E, T2, H
      C(1) =0.0
      C(2) =0.5
      C(3)=0.5
      C(4)=1.0
      DO 4  I=1,4
      TC=T + C(I)* DT
      DO 2 J=1,N
    2 XC(J)= X(J) +C(I)* AK(I-1,J )
      CALL DERIV(TC,XC,XDOT )
      DO 4  J=1,N
    4 AK(I,J)= DT* XDOT(J)
      DO 3 J=1,N
    3 X(J)=X(J)+(AK(1,J)+2.*AK(2,J)+2.* AK(3,J)+AK(4,J))/ 6.
      RETURN
      END
      SUBROUTINE DERIV(T, X, XDOT)
      DIMENSION  XDOT(30), X( 30), H(30)
      COMMON R, E, T2, H
      IF(1.0- T2)3,3,1
    1 R = H(1)
      GO TO 2
    3 R =  -H(1)
      IF(2.0 -T2) 4,4,2
    4 T2= 0.0
      GO TO 1
    2 E = SINF(314.*T)
C     H(4) = VARIABLE POLE
C     H(5) = VARIABLE GAIN
      XDOT(1) = H(5)*X(2)
      XDOT(2) = X(3)*H(3)  - X(2)*H(4)
      XDOT(3) = H(2)*(R - X(1)- X(3))
      XDOT(4) = X(5)*(H(7) - X(7)*H(16))
      XDOT(5) = H(6)*(R - X(5) - X(5)*X(9)*H(17) - X(4))
      XDOT(6) = H(8)*E*ABSF(X(1)- X(4) - E*X(4)) - X(6)*H(9)
      XDOT(7) = H(10)*X(6) - X(7)*H(11)
      XDOT(8) = H(12)*E*ABSF( X(2) - X(5) - X(5)*E) - X(8)*H(13)
      XDOT(9) = H(14)*X(8) - H(15)*X(9)
    7 RETURN
      END
      END
```

Figure 23 a

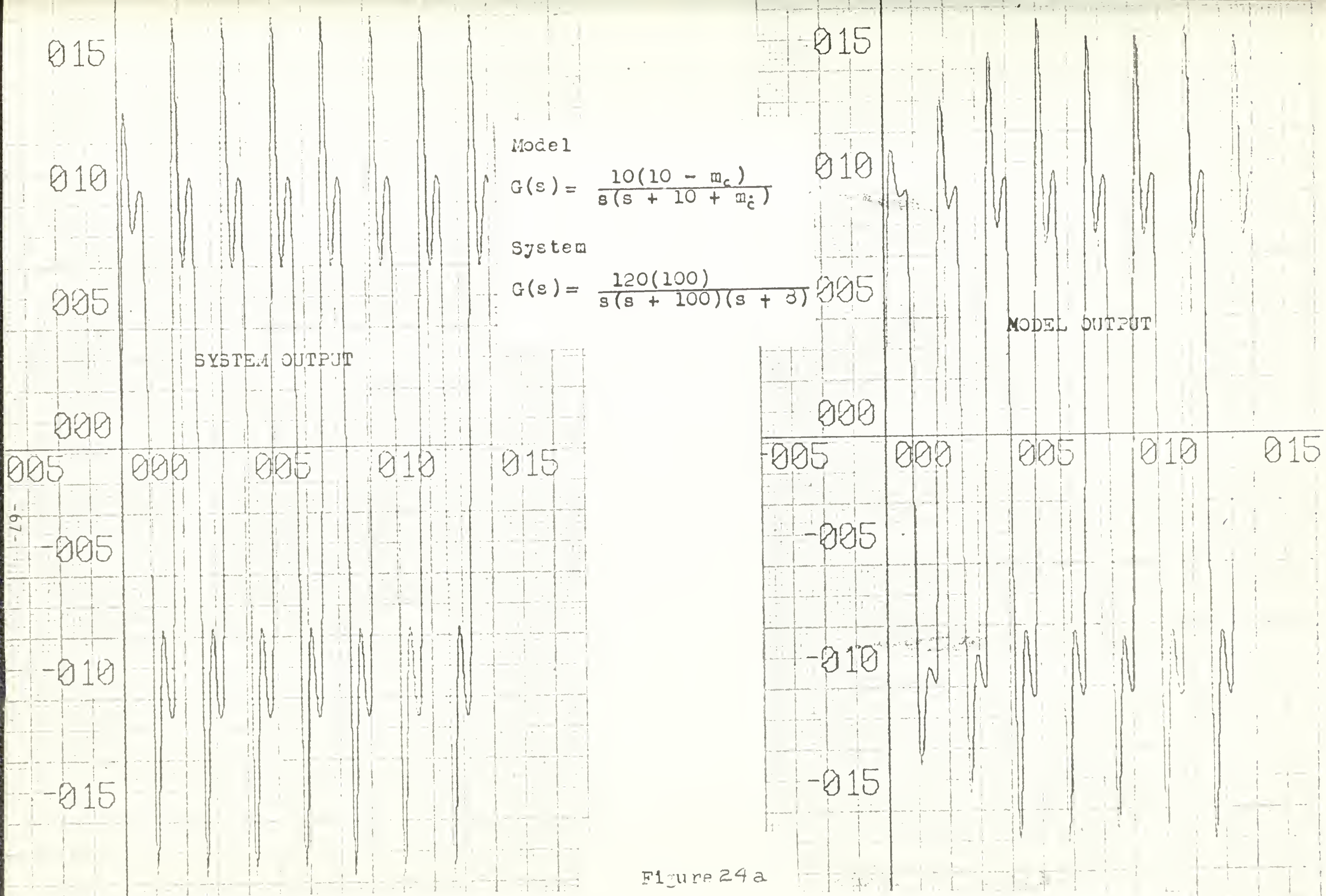
Digital Computer Numerical Output

```
ADAPTIVE SYSTEM GAINS, OUTPUT (S) =  10.00,FIRST DERIV. (D) =  10.00
SYSTEM=   12000.0/S(S+100)(S+   8.00).
GAIN CORR FILTER CORNER AT W =    .10, POLE CORR. FILTER AT W =    .10
MODEL = 100./S(S+10).

COEFFICIENT NUMBER  1 =     10.0000.

COEFFICIENT NUMBER  2 =    100.0000.

COEFFICIENT NUMBER  3 =     10.0000.

COEFFICIENT NUMBER  4 =      8.0000.

COEFFICIENT NUMBER  5 =     12.0000.

COEFFICIENT NUMBER  6 =     10.0000.

COEFFICIENT NUMBER  7 =     10.0000.

COEFFICIENT NUMBER  8 =       .5000.

COEFFICIENT NUMBER  9 =       .5000.

COEFFICIENT NUMBER 10 =       .1000.

COEFFICIENT NUMBER 11 =       .1000.

COEFFICIENT NUMBER 12 =       .5000.

COEFFICIENT NUMBER 13 =       .5000.

COEFFICIENT NUMBER 14 =       .1000.

COEFFICIENT NUMBER 15 =       .1000.

COEFFICIENT NUMBER 16 =     10.0000.

COEFFICIENT NUMBER 17 =     10.0000.
```

TIME	SYS.POS.	MOD.POS.	POS.DIFF.	SYS.VEL.	MOD.VEL.	VEL.DIFF.	GAIN COR.	POLE COR.
.000	.00000	.00000	.00000	.00000	.00000	.00000	10.00000	10.00000
.100	3.69273	3.40275	.28999	5.61750	5.33409	.28341	9.99989	10.00737
.200	10.44149	8.49196	1.94953	4.62161	4.19022	.43139	10.00110	10.00888
.300	13.53981	11.24083	2.29898	.50323	1.33255	-.82932	10.00785	10.00099
.400	12.35534	11.52960	.82575	-1.99107	-.49397	-1.49711	10.02075	10.00606
.500	9.91109	10.74366	-.83257	-1.70828	-.87750	-.83078	10.03537	10.00174
.600	8.73349	10.01976	-1.28627	-.22953	-.50680	.27727	10.04686	9.98130
.700	9.12205	9.74124	-.61919	.70423	-.07426	.77849	10.05401	9.96510
.800	10.00613	9.79000	.21613	.63050	.12882	.50168	10.05885	9.95211
.900	10.45240	9.93120	.52120	.10035	.12843	-.02808	10.06391	9.93724
1.000	10.32668	10.02388	.30279	-.24852	.05299	-.30152	10.07011	9.92324
1.100	2.68922	3.23023	-.54101	-11.44269	-10.68461	-.75808	10.07708	9.92093
1.200	-10.98873	-7.07343	-3.91530	-9.32122	-8.42430	-.89691	10.08451	9.90209
1.300	-17.19482	-12.62957	-4.56525	-.95736	-2.63557	1.67821	10.10174	9.86444
1.400	-14.74071	-13.14862	-1.59210	4.05774	1.07389	2.98385	10.13070	9.85361
1.500	-9.78529	-11.47412	1.68883	3.44661	1.82285	1.62376	10.16267	9.82337
1.600	-7.42477	-9.96699	2.54222	.44240	1.02267	-.58027	10.18814	9.76167
1.700	-8.23215	-9.41829	1.18614	-1.43594	.11295	-1.54889	10.20486	9.71016
1.800	-10.02508	-9.55980	-.46528	-1.27254	-.29882	-.97372	10.21695	9.66551
1.900	-10.92024	-9.87889	-1.04135	-.19509	-.27616	.08107	10.22939	9.61745
2.000	-10.65808	-10.07414	-.58394	.50702	-.10138	.60840	10.24392	9.57255
2.100	-2.69212	-3.10828	.41616	11.67948	10.83759	.84189	10.25954	9.53815
2.200	11.15569	7.54955	3.60615	9.36195	8.52694	.83501	10.27538	9.48632
2.300	17.31802	13.17807	4.13994	.86625	2.45940	-1.59316	10.29984	9.41886
2.400	14.74535	13.42517	1.32017	-4.14494	-1.40254	-2.74240	10.33438	9.37693
2.500	9.72581	11.40898	-1.68317	-3.46370	-2.05385	-1.40985	10.37071	9.31033
2.600	7.37904	9.73783	-2.35879	-.41036	-1.04655	.63619	10.40019	9.21446
2.700	8.22916	9.23581	-1.00665	1.46801	.00155	1.46646	10.42108	9.13227
2.800	10.04623	9.51394	.53229	1.27958	.42065	.85893	10.43752	9.05649
2.900	10.93718	9.93301	1.00417	.18386	.32662	-.14276	10.45415	8.97703
3.000	10.65964	10.14636	.51328	-.51880	.08290	-.60170	10.47243	8.90360
3.100	2.75180	2.92173	-.16993	-11.65724	-11.13523	-.52201	10.49133	8.84414
3.200	-11.10618	-8.32550	-2.78069	-9.39325	-8.77149	-.62176	10.51027	8.77749
3.300	-17.31237	-14.08241	-3.22996	-.90041	-2.19580	1.29538	10.53555	8.70177
3.400	-14.76649	-13.86026	-.90623	4.13600	1.96281	2.17319	10.56822	8.64565
3.500	-9.74411	-11.25895	1.51485	3.47462	2.44851	1.02611	10.60146	8.55695
3.600	-7.38159	-9.32811	1.94652	.42288	1.06956	-.64667	10.62833	8.44594
3.700	-8.22169	-8.93997	.71828	-1.46445	-.21927	-1.24519	10.64780	8.35086
3.800	-10.03948	-9.47315	-.56633	-1.28337	-.63392	-.64945	10.66364	8.25853
3.900	-10.93607	-10.05738	-.87869	-.18844	-.39714	.20870	10.67973	8.16087

Figure 23 b

Digital Computer Numerical Output
(con't)



Model

$$G(s) = \frac{10(10 - m_c)}{s(s + 10 + m_c)}$$

System

$$G(s) = \frac{120(100)}{s(s + 100)(s + 3)}$$

Figure 24 a

Digital Computer Graphical Output for Two Parameter Identification

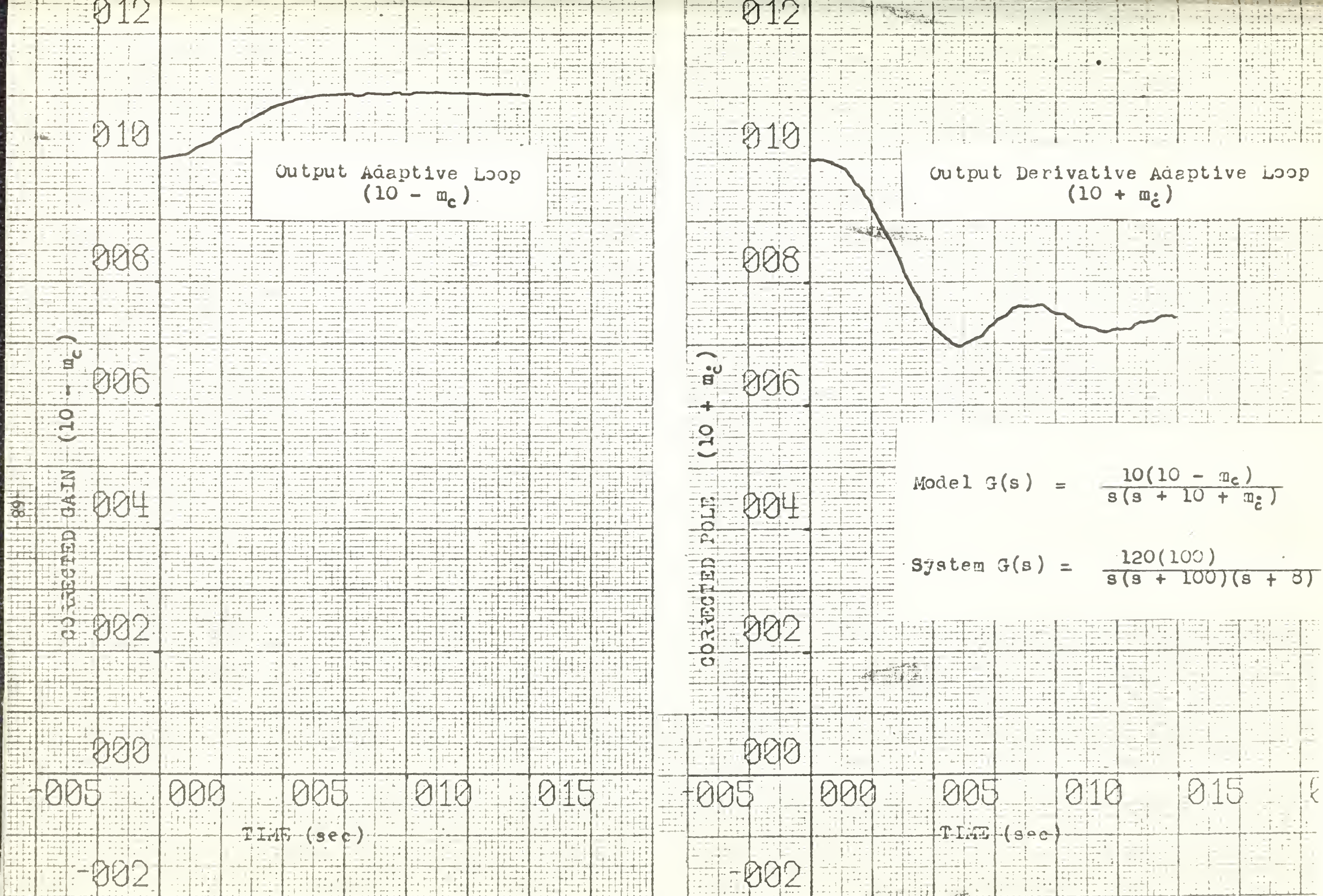


Figure 24 b

Digital Computer Graphical Output for Two Parameter Identification

The third configuration examined was that for the following model open loop transfer function:

$$G(s) = \frac{10,000}{s\ (s + 100 + m_c)\ (s + 10 + m_c)} \qquad 5.6$$

with control system open loop transfer functions of:

$$G(s) = \frac{10,000}{s\ (s + 150)\ (s + 15)} \qquad 5.7$$

and

$$G(s) = \frac{10,000}{s\ (s + 75)\ (s + 7.5)} \qquad 5.8$$

The equations were formulated in the manner of Fig. 20, except that the adaptive loops each utilized one filter and one integrator instead of the two filters previously detailed. Satisfactory results were obtained. Fig. 25 presents sample results from the investigation of equation 5.7, and Table IV presents the root identification achieved.

Table IIIB

DIGITAL COMPUTATION CHARACTERISTIC EQUATION ROOTS

TWO UNCERTAIN POLES

	Control System	Model as Corrected
Equation 5.7	$-7.25 \pm j\,3.72$ -150.49	$-7.87 \pm j\,2.1$ -150.49
Equation 5.8	$-2.81 \pm j\,11.05$ -76.87	$-3.45 \pm j\,11.05$ -74.56

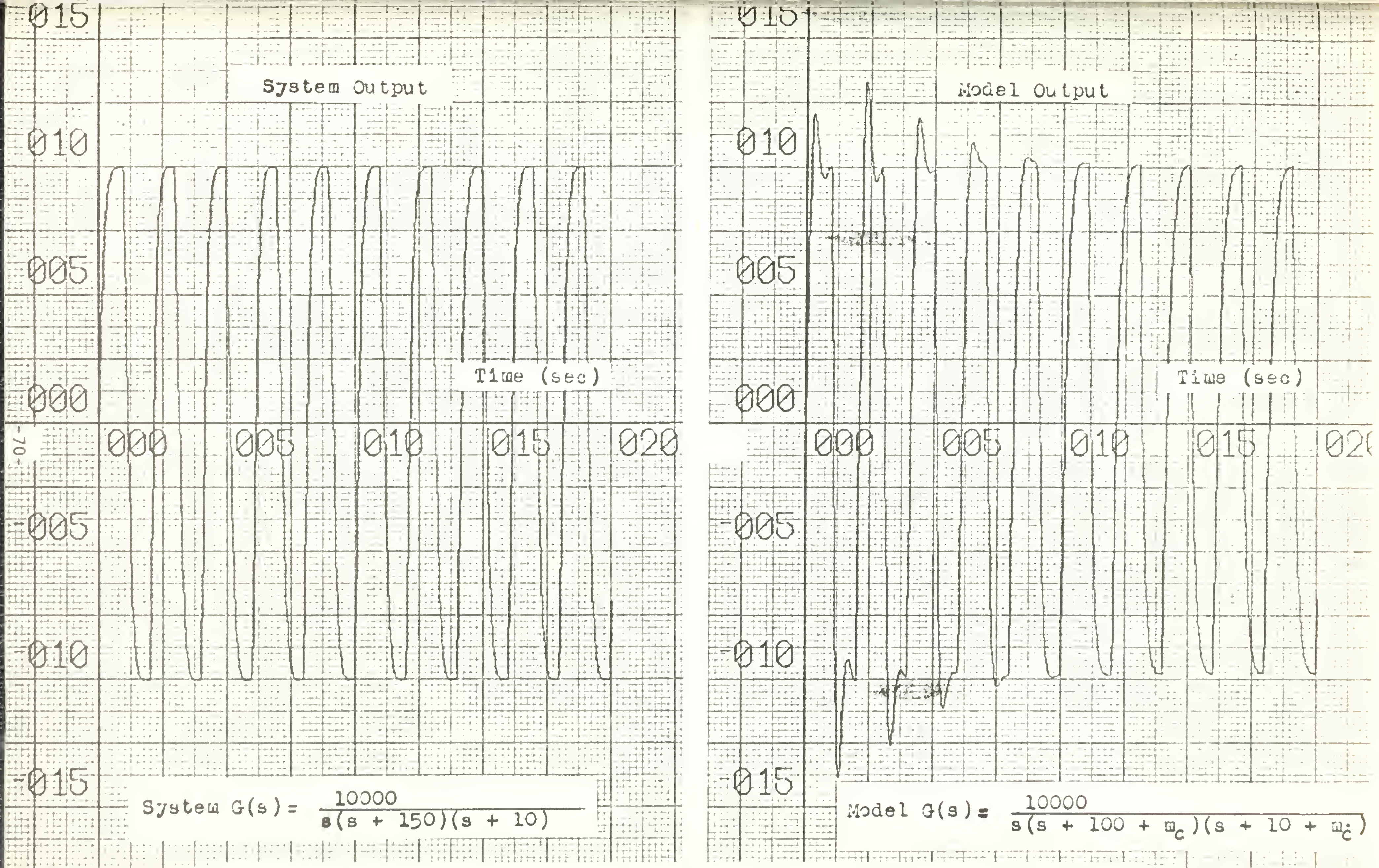


Figure 25a

Digital Computer Graphical Output for Integration in Adaptive Loops

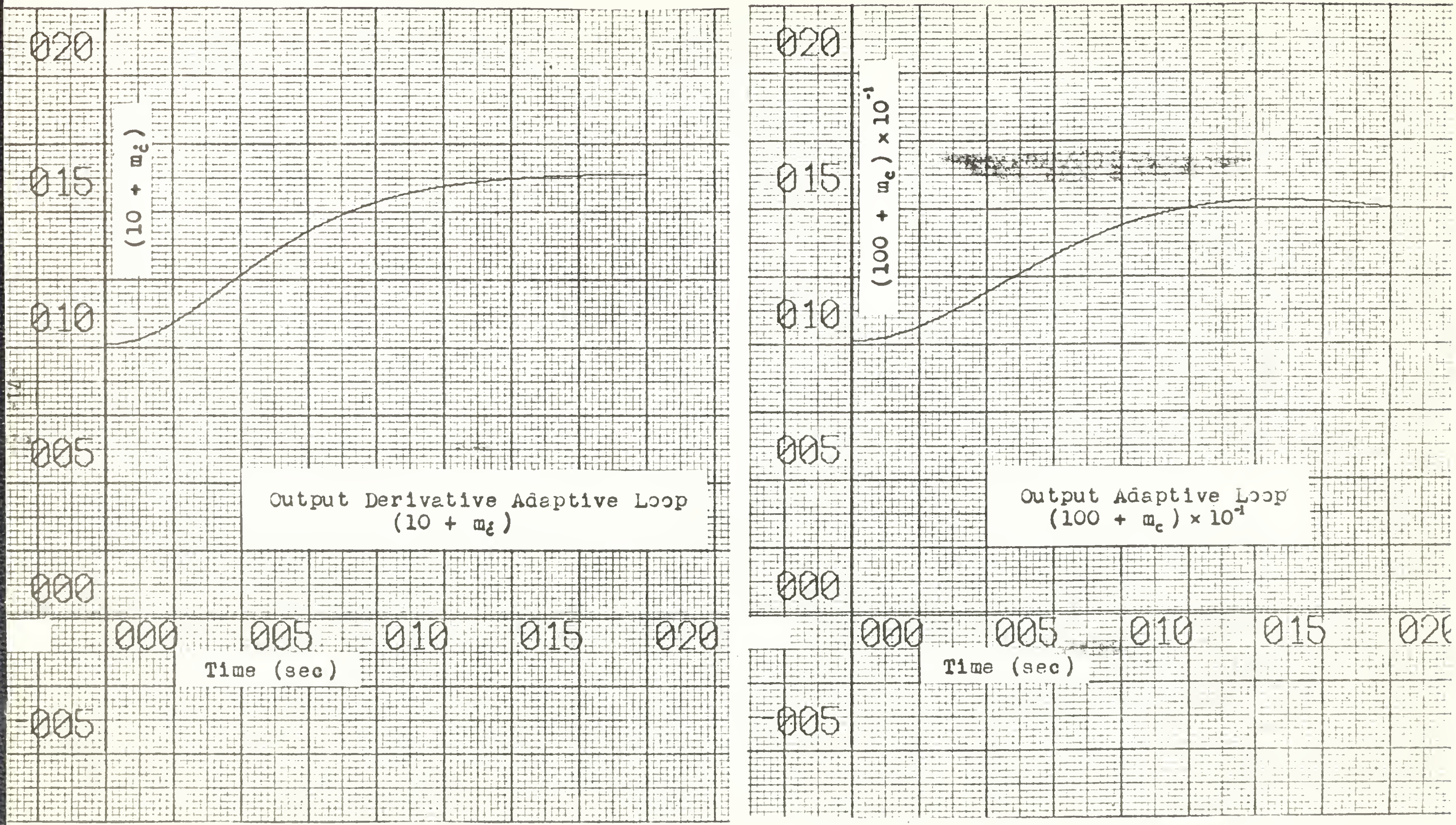


Figure 25b

Digital Computer Graphical Output for Integration in Adaptive Loops

Chapter VI

ACTUAL SERVOMECHANISM IDENTIFICATION

Introduction

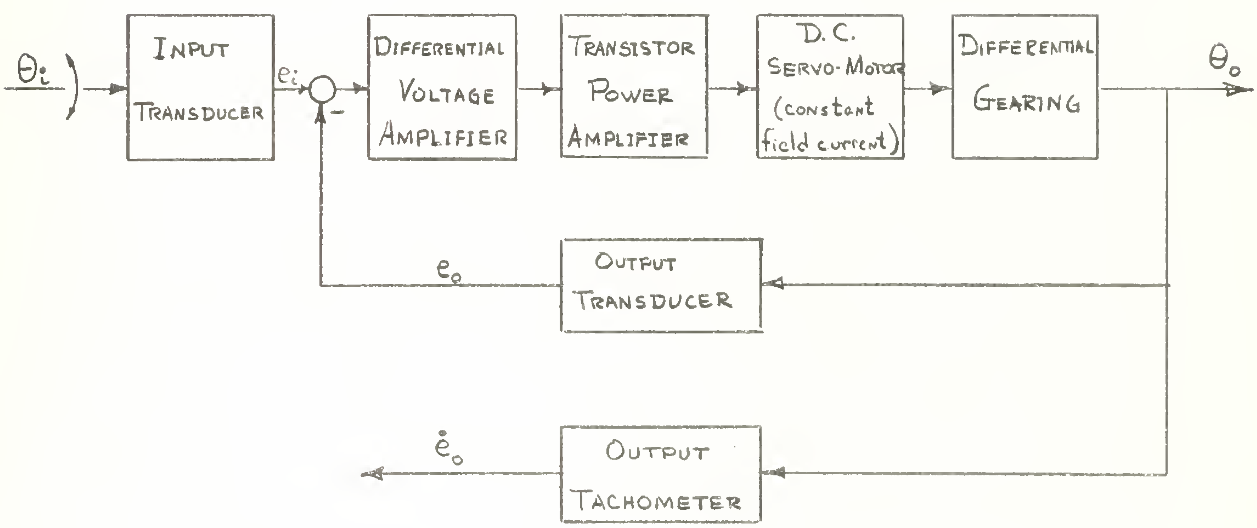
The previous discussion and the data in Table I and Table II indicate that the identified location of the roots of an operating system is the function of many variables. The type of model and the proximity of the model equation roots to the roots of the equation of the operating system are obviously factors of importance in this identification. With this point in mind, it is suggested that the following steps be followed in the process of identifying an actual system.

1. Analyze system operating factors to enable determination of open and closed loop transfer functions.
2. Determine which values in the open loop transfer function are questionable.
3. Utilize operational amplifiers or an analog computer to prepare the dynamic model and the adaptive loops.
4. Scale the model input and outputs to the estimated corresponding values of the system.
5. Set indicated adaptive loop polarities to adapt the model to the undetermined or questionable values in the system.
6. Adjust gain of adaptive loop to obtain satisfactory operation
7. Calculate the transfer function of loop corrected model, scale a new model to these functions and repeat operation to generate better conformance or to confirm values previously attained.

A position servomechanism was used to illustrate the applicability of the model modulation techniques to an actual operating system. The servo used was one developed for use in instructional laboratories by Professor J. R. Ward of the U. S. Naval Postgraduate School, and is partially described in Ref. 6. The full description and details of the operation of this system are on file at this school.

Fig. 26 presents a block diagram of the system. This representation shows that the servomechanism may be considered as a gear-head system. Thus the use of this identification method conforms to the stated requirement of parallel model operation.

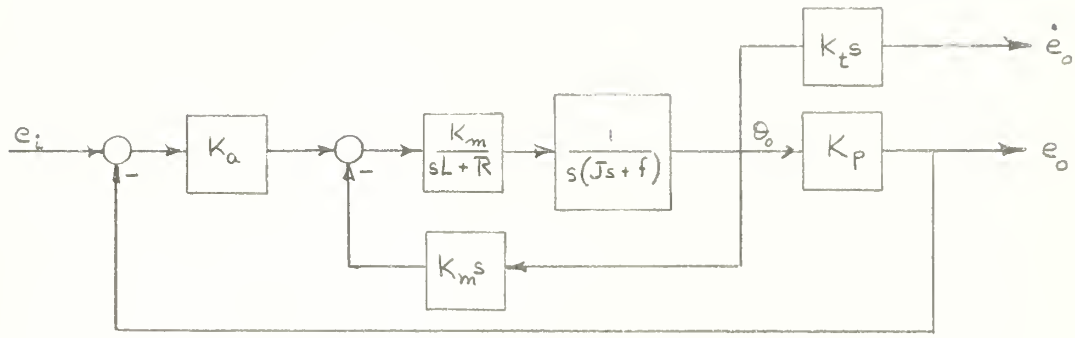
Fig. 26
POSITION SERVOMECHANISM



Analysis of this mechanism resulted in the transfer function diagram of Fig 27.

Fig. 27

POSITION SERVOMECHANISM BLOCK DIAGRAM



The approximate parameter values in the system were determined. The magnitude of inductance (L) was such that the approximation $L \to 0$ was made. The open loop transfer function was then:

$$\frac{e_o}{\mathcal{E}} = G(s) = \frac{\frac{K_a K_p K_m}{RJ}}{s\left(s + \frac{Rf + K_m^2}{RJ}\right)} = \frac{k}{s(s+p)} \qquad 6.1$$

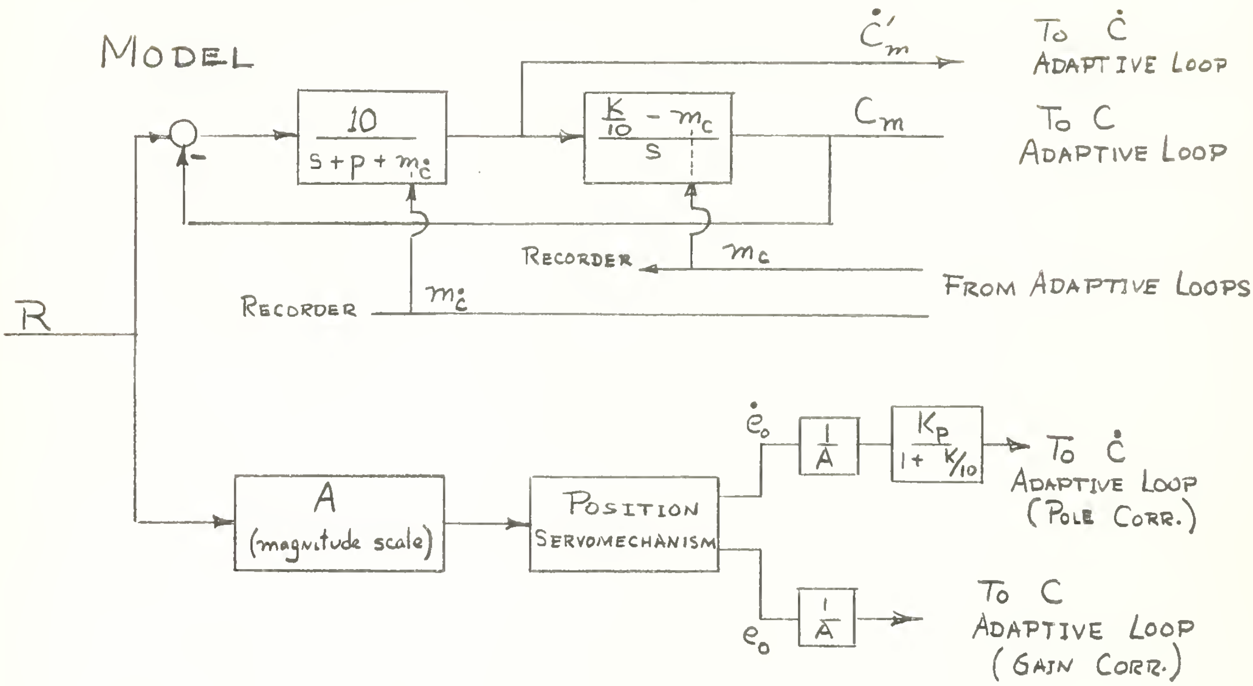
The value of friction(f) was difficult to evaluate because of the expected non-linearity of friction in a d.c. motor. A nominal value was chosen to represent the viscous friction, while coulomb friction effects were ignored. It was further assumed that the open loop gain was questionable. As a first estimate it was assumed that the open loop transfer function was:

$$G(s) = \frac{k}{s(s+p)} \cong \frac{400}{s(s+8)} \qquad 6.2$$

Fig. 28 presents the control system-model-adaptive loop block diagram.

Fig. 28

SERVOMECHANISM IDENTIFICATION BLOCK DIAGRAM



The actual model and adaptive loop analog computer schematic was that detailed in Fig. 15, while the scaling factors were achieved through cascaded operational amplifiers to assure correct polarity. The servomechanism and the model were driven by a square wave of one cycle per second. Table IV presents the results of four tests made first to bracket, then to closely approximate the open loop transfer function. Fig. 29 presents the results of the final run listed in Table IV. One the basis of these results the transfer function of this servomechanism was estimated to be:

$$\frac{e_o(s)}{e_i(s)} = \frac{625}{s^2 + 10.15s + 625} \qquad 6.3$$

These results compare favorably with information derived from other sources. Calculated results derived for careful component analysis

Table IV

POSITION SERVOMECHANISM CHARACTERISTIC EQUATION IDENTIFICATION

OPEN LOOP TRANSFER FUNCTION		SYSTEM DYNAMIC EQUATION
Model	Corrected Model	Identified Roots
$\frac{10\ (40-m_c)}{s\ (s+8+m_c)}$	$\frac{574}{s\ (s+9.66)}$	$-4.83 \pm j\ 23.45$ $\zeta = .201 \quad \omega_n = 24$
$\frac{10\ (70-m_c)}{s\ (s+10+m_c)}$	$\frac{652}{s\ (s+10.15)}$	$-5.075 \pm j\ 25$ $\zeta = .204 \quad \omega_n = 25.55$
$\frac{10\ (55-m_c)}{s\ (s+10+m_c)}$	$\frac{622}{s\ (s+10.24)}$	$-5.12 \pm j\ 24.2$ $\zeta = .206 \quad \omega_n = 24.95$
$\frac{10\ (60-m_c)}{s\ (s+10+m_c)}$	$\frac{625}{s\ (s+10.15)}$	$-5.075 \pm j\ 24.4$ $\zeta = .208 \quad \omega_n = 25$

indicated a closed loop transfer function of

$$\frac{e_o(s)}{e_i(s)} = \frac{590}{s^2 + (12.5 \pm 1.5)s + 590} \qquad 6.4$$

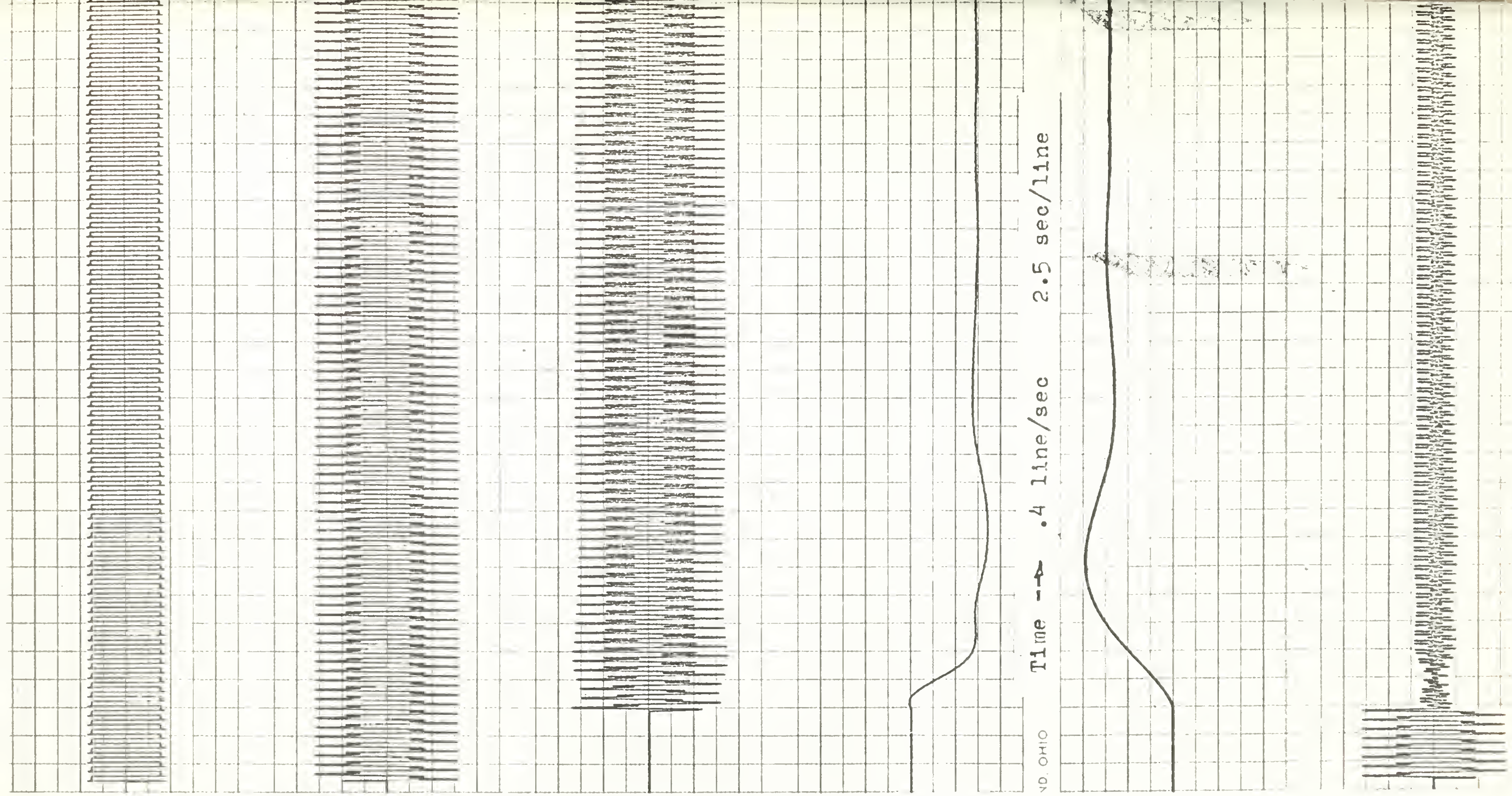
while various open loop frequency response tests indicated the closed loop response to be:

$$\frac{e_o(s)}{e_i(s)} = \frac{565 \pm 70}{s^2 + (10 \pm 1)s + 565 \pm 70} \qquad 6.5$$

In view of the non-linearities involved in a d.c. servo motor it was felt that these model identified roots represented a successful illustration of the validity of the method.

Discussion

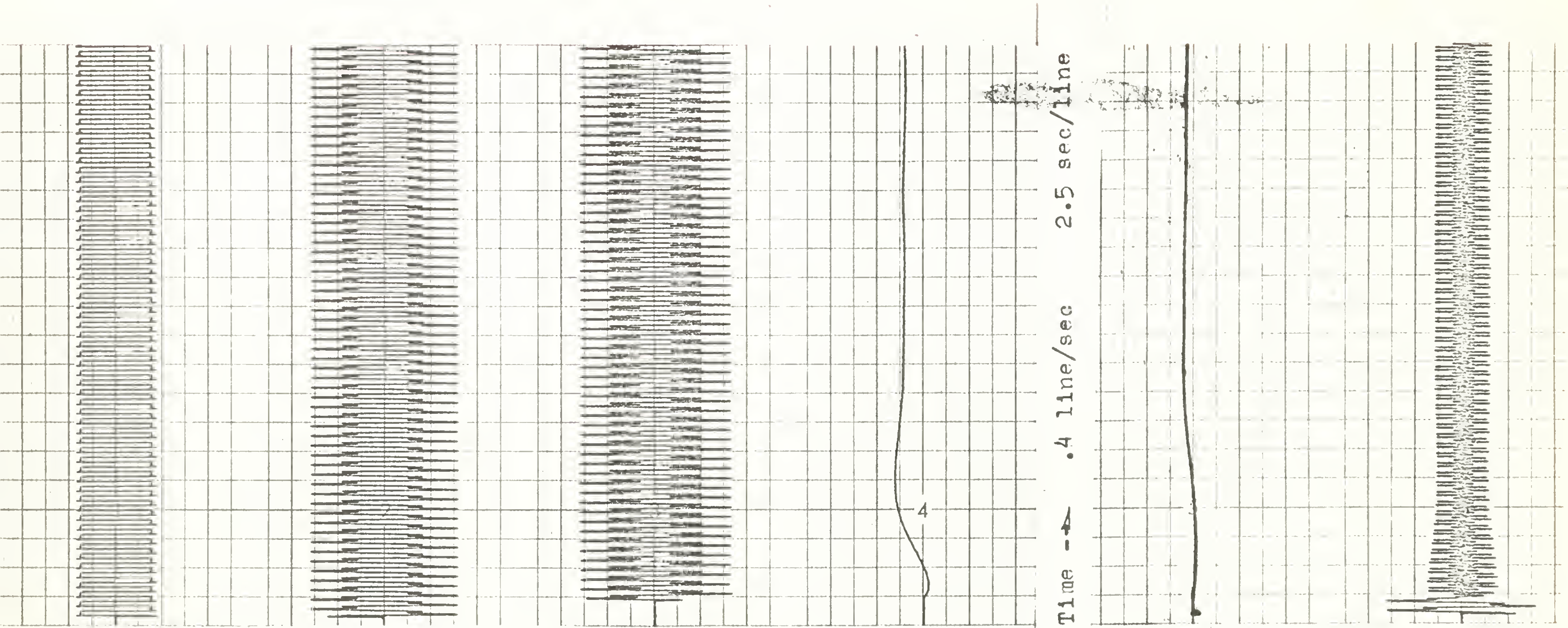
These servomechanism tests confirmed that the model modulation method could be successfully applied to obtain the governing operating equations of a servomechanism. Beyond this the tests emphasized the necessity of continuous excitation of the control system for successful model adaptation. For a linear control system the model modulation adaptation will operate only while the servo is undergoing excitated or active response to an input signal. It would be expected that the correction or adaptive signal would be generated in response to a step for a period equal to the settling time of the system. Beyond the settling time, with no excitation response operation to generate error, the time constants of the adaptive filter network will allow dissipation of the adaptive signal magnitude. The analog computer simulations detailed in this chapter illustrate this operation of the



Model $G(s) = \dfrac{10(40 - m_c)}{s(s + 8 + m_\varepsilon)}$ Identified Servo $G(s) = \dfrac{574}{s(s + 9.66)}$

Figure 29a

Servomechanism Identification



Input
2 v/1

Servomechanism
Output 2 v/1

Model
2 v/1

Gain Adaptive
Loop (m_c)
0.12 per line

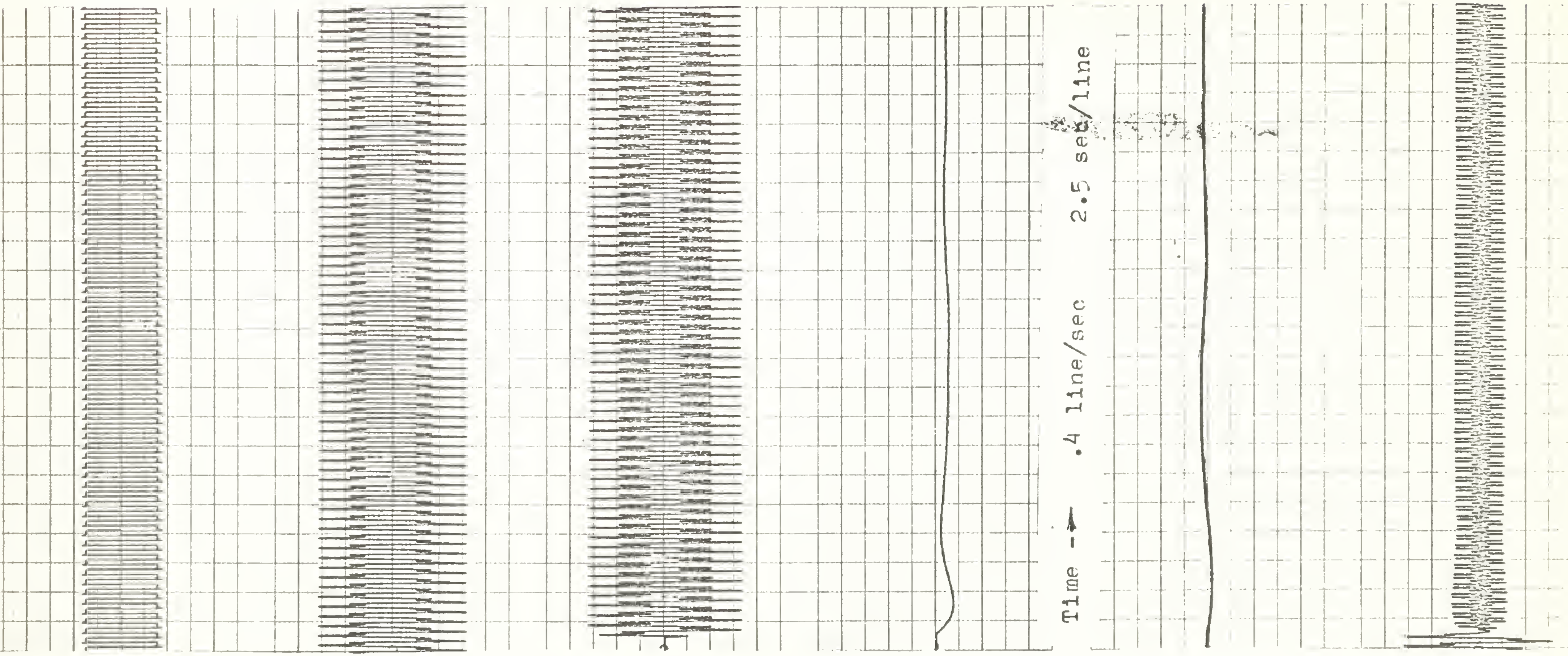
Pole Adaptive
Loop (m_c')
0.12 per line

$C_m - C_s$
1 v/1

$$\text{Model } G(s) = \frac{10(70 - m_c)}{s(s + 10 + m_c')} \qquad \text{Identified Servo } G(s) = \frac{652}{s(s + 10.15)}$$

Figure 29b

Servomechanism Identification



$$\text{Model } G(s) = \frac{10(60 - m_c)}{s(s + 10 + m_{\dot c})} \qquad \text{Identified Servo } G(s) = \frac{625}{s(s + 10.15)}$$

Figure 29 c

Servomechanism Identification

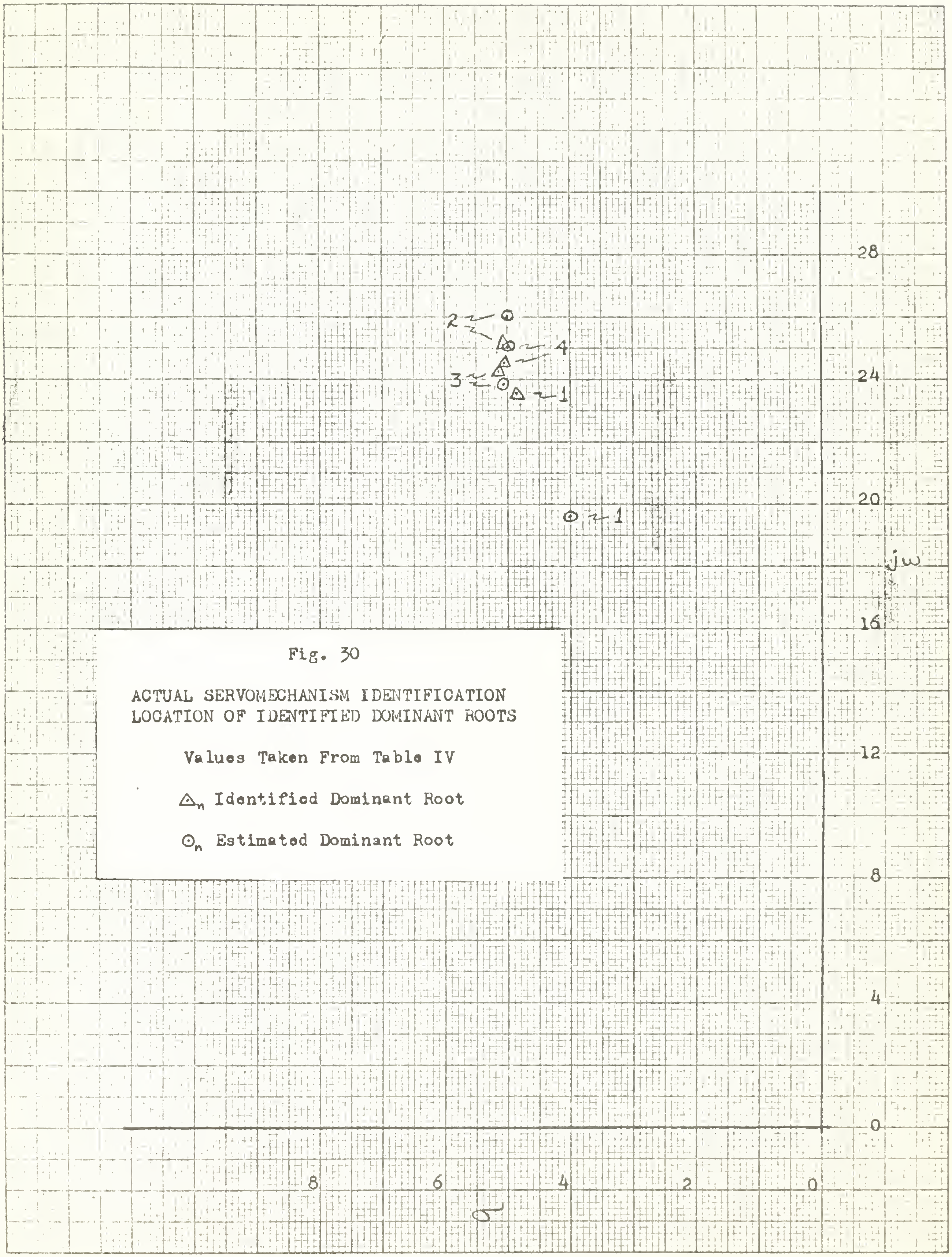


Fig. 30

ACTUAL SERVOMECHANISM IDENTIFICATION
LOCATION OF IDENTIFIED DOMINANT ROOTS

Values Taken From Table IV

$\triangle_n$ Identified Dominant Root

$\odot_n$ Estimated Dominant Root

adaptive loop For the analog tests the general input frequency of the square wave was 0.5 cps. The average settling time for the second order model was $\tau \cong 4/\delta\omega_n \simeq 1.0$ sec, and the bandwidth was $f \simeq 1.9$ cps. When tests were run with systems with small zeta, lower input frequencies would achieve successful adaptation. In all cases, successful operation was attained with frequencies which were as high as the bandwidth frequency.

In the case of the operating servo, the non-linearities of the friction caused a faster damping than the linear interpolation would indicate. For this system, the adaptation proceeded successfully when the driving square wave frequency allowed limited oscillation per step. This in effect was a partial masking of the non-linearity through testing techniques.

These situations emphasize the limitation of control system excitation in the use of the model modulation method. It is obvious that this limitation exists for both system adaptation and for system identification through model adaptation. A potential solution of this problem might be found through investigation into utilization of signal identification circuits. This signal identification might be applied to the output of the control system to control the operation of the adaptive circuit. This identifier could allow the adaptive circuit to operate while the control system would be underdoing oscillatory or excited response. When this control system response became quiescent, the identifier could place the adaptive loop into a hold state with a constant level of output maintained to maintain

the adaptation until such time as the control system would be again activated. Additionally, it is expected that the adaptive loop elements would be synthesized through the use of operational amplifiers. Since these are magnitude sensitive, another role of the signal identifier would be to hold adaptive loop operation when signal magnitude became excessive.

The techniques of sampled-data control systems offer attractive possibilities for circuits to fulfill these requirements. Hybrid digital-analog circuits might also be investigated. In any case, these techniques warrant additional investigation.

Chapter VII

CONCLUSIONS AND RECOMMENDATIONS

Conclusions

It is concluded from the results of this investigation that the identification of the characteristic equation roots of a control system by model adaptation through model modulation is a valid technique. The technique is subject to operating restrictions as follows:

1. The control system must be linear or be subjected to operating non-linearities of minor magnitude.
2. The open loop transfer function must have but one or two parameters whose magnitude is uncertain.
3. The additional parameters of the open loop transfer function must be known or be of sufficient magnitude to be inconsequential.
4. The control system must be undergoing active operation in response to an input signal.

It is further concluded that:

1. The roots of the characteristic equation of a control system may be identified with a high degree of accuracy. During these tests, these dominant roots were identified to within an average error of 2.2% of the undamped natural frequency and to within an average error of 4.76% of the complex phase angle.

2. When the order of the characteristic equation is unknown, second order dominant mode approximations will yield consistent results which are valid in the approximation.

Recommendations

The results of this investigation indicate that there are certain areas of interest which warrant further investigation. The first of these is investigation into the stability of the dynamic adaptive loop. The stability limits are subject to various major bounds in loop gain, input signal frequency, and input signal magnitude. Theoretical analysis of these bounds will be extremely difficult, thus analog computer investigation might offer a valid method of investigation.

The second of these would be the investigation of methods to remove the necessity of a continual excited response from the control system. Certain suggestions have been made in the text which might be applied to achieve this end.

The third suggested area of investigation is in the identification of control system operation subject to known non-linearities.

REFERENCES

Ref. 1 Mishkin, Eli, and Braun, Ludwig, Jr., ADAPTIVE CONTROL SYSTEMS, 1961, McGraw-Hill Book Company, Inc., New York

Ref. 2 Chang, Sheldon, S. L., SYNTHESIS OF OPTIMUM CONTROL SYSTEMS, 1961, McGraw-Hill Book Company, Inc., New York

Ref. 3 A Parameter-Perturbation Adaptive Control System, McGrath, R. J., Rajaraman, V., Rideout, V. C., Transactions I.R.E. Vol. AC7, May 1961.

Ref. 4 Theory of a Two Parameter Adaptive Control System, Rajaraman, V., Transactions I.R.E., Vol. AC7-#4, July, 1962.

Ref. 5 A Modulation Method for Model Reference Adaptive Control Systems, Dorf, R. C., and Byers, W. A., Proceedings, IEEE International Convention Record, to be published July 1963.

Ref. 6 An Instructional Servomechanisms Laboratory, Ward, J. R., I.R.E. Transactions on Education, Vol E-5, March 1962.